16 GOVERNORS

Introduction

The function of a governor is to maintain the speed of an engine within specified limits whenever there is a variation of load. In general, the speed of an engine varies in two ways—during each revolution (cyclic variation) and over a number of revolutions. In the former case, it is due to variation in the output torque of the engine during a cycle and can be regulated by mounting a suitable flywheel on the shaft. In the latter case, it is due to variation of load upon the engine and requires a governor to maintain the speed. The operation of a flywheel is continuous whereas that of a governor is more or less intermittent. A flywheel may not be used if there is no undesirable cyclic fluctuation of the energy output, but a governor is essential for all types of engines as it adjusts the supply according to the demand.

If the load on the shaft increases, the speed of the engine decreases unless the supply of fuel is increased by opening the throttle valve. On the other hand, if the load on the shaft decreases, the speed of the engine increases unless the fuel supply is decreased by closing the valve sufficiently to slow the engine to its original speed. The throttle valve is operated by the governor through a mechanism for the purpose.

16.1 TYPES OF GOVERNORS

Governors can broadly be classified into two types.

(i) Centrifugal Governor

This is the more common type. Its action depends on the change of speed. It has a pair of masses, known as governor balls, which rotate with a spindle. The spindle is driven by an engine through bevel gears (Fig. 16.1). The action of the governor depends upon the centrifugal effects produced by the masses of the two balls. With the increase in the speed, the balls tend to rotate at a greater radius from the axis. This causes the sleeve to slide up on the spindle and this movement of the sleeve is communicated to the throttle through a bell crank lever. This closes the throttle valve to the required

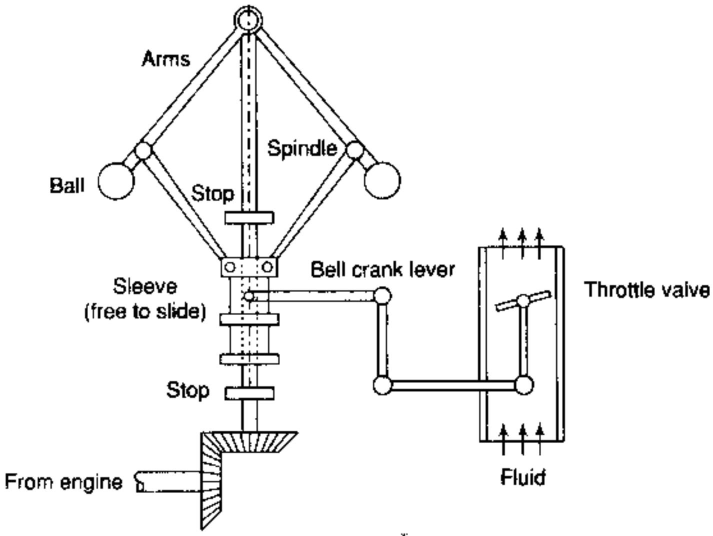


Fig. 16.1

extent. When the speed decreases, the balls rotate at a smaller radius and the valve is opened according to the requirement.

(ii) Inertia Governor

In this type, the positions of the balls are affected by the forces set up by an angular acceleration or deceleration of the given spindle in addition to centrifugal forces on the balls. Using suitable linkages and springs, the change in position of the balls is made to open or close the throttle valve.

Thus, whereas the balls are operated by the actual change of engine speed in the case of centrifugal governors, it is by the rate of change of speed in case of inertia governors. Therefore, the response of inertia governors is faster than that of centrifugal types.

WATT GOVERNOR (SIMPLE CONICAL GOVERNOR)

Figure 16.2 shows three forms of a simple centrifugal or a Watt governor. In this, a pair of balls (masses) is attached to a spindle with the help of links. In Fig.16.2(a), the upper links are pinned at point O. In the Fig.16.1(b), the upper links are connected by a horizontal link and the governor is known as the *open-arm* type Watt governor. On extending the upper arms, they still meet at O. In Fig.16.2(c), the upper links cross the spindle and are connected by a horizontal link and the governor is known as a *crossed-arm* Watt governor. In this type also, the two links intersect at O. The lower links in every case are fixed to a sleeve free to move on the vertical spindle.

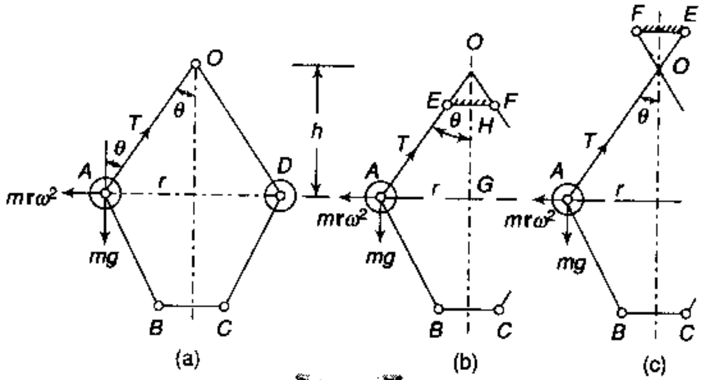


Fig. 16.2

As the spindle rotates, the balls take up a position depending upon the speed of the spindle. If it lowers, they move near to the axis due to reduction in the centrifugal force on the balls and the ability of the sleeve to slide on the spindle. The movement of the sleeve is further taken to the throttle of the engine by means of a suitable linkage to decrease or increase the fuel supply.

The vertical distance from the plane (horizontal) of rotation of the balls to the point of intersection of the upper arms along the axis of the spindle is called the *height of the governor*. The height of the governor decreases with increase in speed, and increases with decrease in speed.

Let m = mass of each ball

h = height of governor

w = weight of each ball (= mg)

ω = angular velocity of the balls, arms and the sleeve

T = tension in the arm

r = radial distance of ball-centre from spindle-axis

Assuming the links to be massless and neglecting the friction of the sleeve, the mass m at A is in static equilibrium under the action of

- Weight **w** (= mg)
- Centrifugal force $mr\omega^2$
- Tension **T** in the upper link

If the sleeve is massless and also friction is neglected, the lower links will be tension free.

The equilibrium of the mass provides

$$T\cos\theta = mg \text{ and } T\sin\theta = mr\omega^2$$

$$\therefore \qquad \tan\theta = \frac{mr\omega^2}{mg} = \frac{r\omega^2}{g}$$

or

$$\frac{r}{h} = \frac{r\omega^2}{g}$$

or

$$h = \frac{g}{\omega^2} = \frac{g}{\left(\frac{2\pi N}{60}\right)} = \left(\frac{60}{2\pi}\right)^2 \times \frac{9.81}{N^2}$$

$$= \frac{895}{N^2}\text{ m} \tag{16.1}$$

or

$$h = \frac{895\ 000}{N^2}\text{ mm}$$

Thus, the height of a Watt governor is inversely proportional to the square of the speed. A close look at this equation would reveal that the variation in h is appreciable for low values of speed N. As the speed N becomes larger, the variation in h becomes very small.

The following table shows the height h with the variation in speed:

N (rpm)	50	100	150	200	300	400
h (mm)	358	89.5	39.8	22.4	9.9	5.6

This shows that in this type of governor, the movement of the sleeve is very less at high speeds and thus is unsuitable for these speeds. However, this drawback has been overcome by loading the governor with a dead weight or by means of a spring. Such governors have been discussed in the sections that follow.

Example 16.1 *In an open-arm type governor [Fig. 16.3(a)], AE = 400 mm, EF = 50 mm and angle θ = 35°. Determine the percentage change in speed when θ decreases to 30°.*

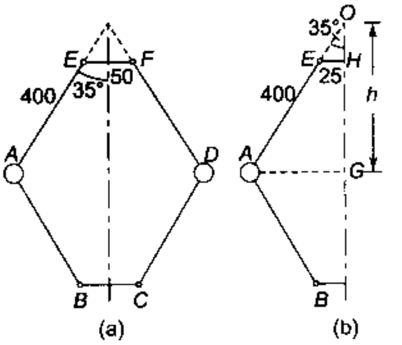


Fig. 16.3

Solution Refer Fig. 16.3(b),

$h = GO = GH + HO = AE\cos\theta + EH\cot\theta$

$h = 400\cos 35° + 25\cot 35°$

$= 363.4$ mm

$h' = 400\cos 30° + 25\cot 30°$

$= 389.7$ mm

Now, $h = \dfrac{g}{\omega^2}$ and $h' = \dfrac{g}{\omega'^2}$

$$\therefore \qquad \frac{\omega'}{\omega} = \sqrt{\frac{h}{h'}} = \sqrt{\frac{363.4}{389.7}} = 0.966$$

Decrease in speed = $(1 - 0.966) \times 100 = \underline{3.44\%}$

Alternatively,

$$N = \sqrt{\frac{895000}{h}} = \sqrt{\frac{895000}{363.4}} = 49.63 \text{ rpm}$$

$$N' = \sqrt{\frac{895000}{389.7}} = 47.92 \text{ rpm}$$

$$\text{Decrease} = \frac{N - N'}{N} \times 100 = \frac{49.63 - 47.92}{49.63} = \underline{3.44\%}$$

16.3 PORTER GOVERNOR

If the sleeve of a Watt governor is loaded with a heavy mass, it becomes a Porter governor [Fig.16.4(a)]

Let M = mass of the sleeve

m = mass of each ball

f = force of friction at the sleeve

The force of friction always acts in a direction opposite to that of the motion. Thus when the sleeve moves up, the force of friction acts in the downward direction and the downward force acting on the sleeve is $(Mg + f)$. Similarly, when the sleeve moves down, the force on the sleeve will be $(Mg - f)$. In general, the net force acting on the sleeve is $(Mg \pm f)$ depending upon whether the sleeve moves upwards or downwards.

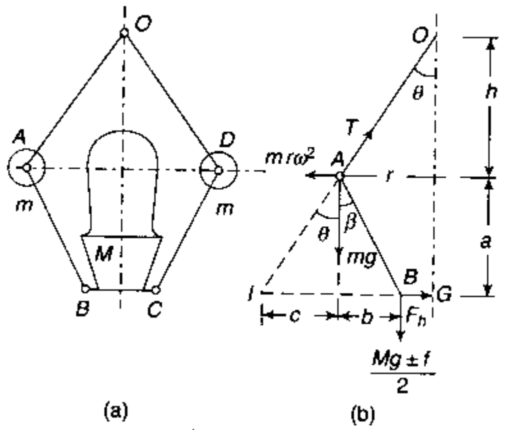


Fig. 16.4

Forces acting on the sleeve and on each ball have been shown in Fig.16.4(b).

Let h = height of the governor

r = distance of the centre of each ball from axis of rotation

The instantaneous centre of rotation of the link AB is at I for the given configuration of the governor. It is because the motion of its two points A and B relative to the link is known. The point A oscillates about the point O and B moves in a vertical direction parallel to the axis. Lines perpendicular to the direction of these motions locates the point I.

Considering the equilibrium of the left-hand half of the governor and taking moments about I,

$$mr\omega^2 . a = mg\,c + \frac{Mg \pm f}{2}(c + b)$$

or $$mr\omega^2 = mg\frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a} + \frac{b}{a}\right)$$

$$= mg\tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)$$

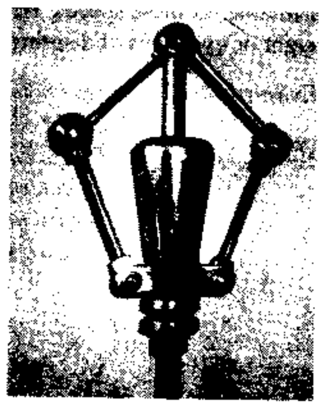
$$= \tan\theta\left[mg + \frac{Mg \pm f}{2}(1 + k)\right] \qquad \left(\text{taking } k = \frac{\tan\beta}{\tan\theta}\right)$$

or $$= \frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1 + k)\right]$$

or $$\omega^2 = \frac{1}{mh}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2}\right)$$

or $$\left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right)$$

$$N^2 = \frac{895}{h}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right) \qquad (\text{Taking } g = 9.81 \text{ m/s}^2)$$



A porter governor

This equation would provide two values of N for the same height of the governor. The phenomenon can be explained as below.

First assume that the sleeve has just moved down. This means that the force acting on the sleeve is $(Mg - f)$ downwards. Now, if the speed of the engine increases, the balls would tend to move away from the axis, but now as the friction has to act in the downward direction, the resistance to the motion would be $(Mg + f)$. Thus until the speed rises to such a value as to overcome this resistance, the sleeve will not move. In the same way, when the sleeve has moved up and the speed decreases, the resistance to the sleeve movement would be only $(Mg - f)$. Thus, until the speed reduces to such a value as to give a force equal to $(Mg - f)$, the sleeve will not move.

Thus, for a given value of h, the governor is insensitive between two values of ω given by Eq. (16.3).

- If $k = 1$, $$N^2 = \frac{895}{h}\left(\frac{mg + (Mg \pm f)}{mg}\right)$$
- If $f = 0$, $$N^2 = \frac{895}{h}\left(\frac{2m + M(1+k)}{2m}\right)$$
- If $k = 1, f = 0$ $$N^2 = \frac{895}{h}\left(\frac{m + M}{m}\right)$$

Example 16.2 *Each arm of a Porter governor is 200 mm long and is pivoted on the axis of the governor. The radii of rotation of the balls at the minimum and the maximum speeds are 120 mm and 160 mm respectively. The mass of the sleeve is 24 kg and each ball is 4 kg. Find the range of speed of the governor. Also determine the range of speed if the friction at the sleeve is 18 N.*

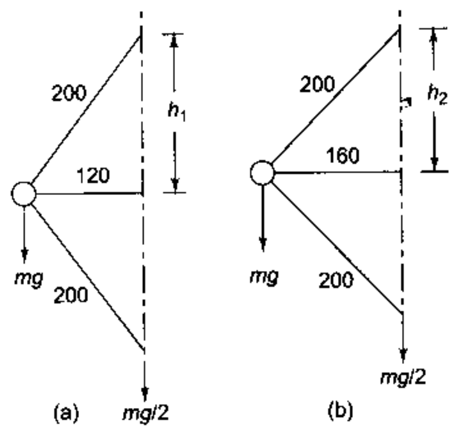


Fig. 16.5

Solution $m = 4$ kg, $M = 24$ kg, $f = 18$ N

At minimum speed, $h = \sqrt{200^2 - 120^2} = 160$ mm [Fig. 16.5(a)]

As $k = 1, f = 0$,

$$N^2 = \frac{895}{h}\left(\frac{m+M}{m}\right) = \frac{895}{0.16}\left(\frac{4+24}{4}\right) = 39\ 156$$

or $N = 197.9$ rpm

At maximum speed, $h = \sqrt{200^2 - 160^2} = 120$ mm [Fig. 16.5(b)]

As $k = 1, f = 0$,

$$N^2 = \frac{895}{h}\left(\frac{m+M}{m}\right) = \frac{895}{0.12}\left(\frac{4+24}{4}\right) = 52\ 208$$

or $N = 228.5$ rpm

Range of speed = 228.5 − 197.9 = 30.6 rpm

When friction at the sleeve is 18 N

At minimum speed,

$$N^2 = \frac{895}{h}\left(\frac{mg + (Mg - f)}{mg}\right)$$

$$= \frac{895}{0.16}\left(\frac{4\times 9.81 + (24\times 9.81 - 18)}{4\times 9.81}\right)$$

$= 36\ 590$ or $N = 191.3$ rpm

At maximum speed,

$$N^2 = \frac{895}{h}\left(\frac{mg + (Mg + f)}{mg}\right)$$

$$= \frac{895}{0.12}\left(\frac{4\times 9.81 + (24\times 9.81 + 18)}{4\times 9.81}\right)$$

$= 55\ 630$ or $N = 235.9$ rpm

Range of speed = 235.9 – 191.3 = 44.6 rpm

Example 16.3 ***In a Porter governor, each of the four arms is 400 mm long. The upper arms are pivoted on the axis of the sleeve, whereas the lower arms are attached to the sleeve at a distance of 45 mm from the axis of rotation. Each ball has a mass of 8 kg and the load on the sleeve is 60 kg. What will be the equilibrium speeds for the two extreme radii of 250 mm and 300 mm of rotation of the governor balls?***

Solution Refer Fig. 16.6,

$m = 8$ kg $\quad BG = 45$ mm

$M = 60$ kg $\quad OA = 400$ mm

We have,

$$mr\omega^2 = \tan\theta\left[mg + \frac{mg \pm f}{2}(1+k)\right] \quad (f = 0)$$

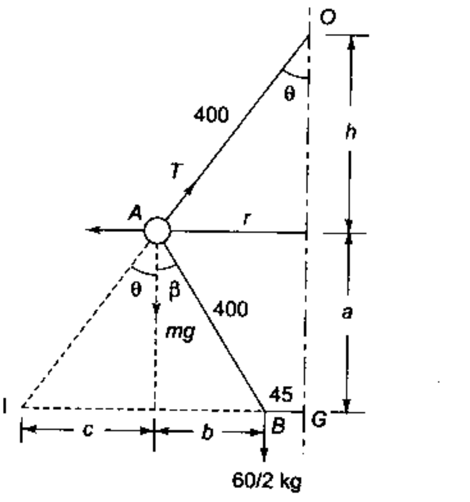


Fig. 16.6

(i) When $r = 250$ mm

$$\tan\theta = \frac{r}{h} = \frac{r}{\sqrt{(OA)^2 - r^2}}$$

$$= \frac{250}{\sqrt{(400)^2 - (250)^2}} = 0.8$$

$$k = \frac{\tan\beta}{\tan\theta} = \frac{b/a}{\tan\theta}$$

As $b = 250 - 45 = 205$ mm,

$$a = \sqrt{(AB)^2 - (b)^2}$$

$$= \sqrt{(400)^2 - (205)^2} = 343.4 \text{ mm}$$

$$k = \frac{205/343.4}{0.8} = 0.746$$

$$\therefore \quad 8\times 0.25\times\omega^2 =$$

$$0.8\left[8\times 9.81 + \frac{60\times 9.81}{2}(1+0.746)\right]$$

$2\omega^2 = 0.8\,(78.48 + 513.85)$

$\omega^2 = 237$

$$\omega = \frac{2\pi N}{60} = 15.39$$

$N = \underline{147 \text{ rpm}}$

(ii) When $r = 300$ mm,

$$\tan\theta = \frac{300}{\sqrt{(400)^2 - (300)^2}} = 1.134$$

$b = 300 - 45 = 255$ mm

$$a = \sqrt{(400)^2 - (255)^2} = 308.2 \text{ mm}$$

$$k = \frac{\tan\beta}{\tan\theta} = \frac{b/a}{\tan\theta} = \frac{255/308.2}{1.134} = 0.73$$

$$\therefore 8\times 0.3\times\omega^2 =$$

$$1.134\left[8\times 9.81 + \frac{60\times 9.81}{2}(1+0.73)\right]$$

$2.4\omega^2 = 1.134\,(78.48 + 509.14)$

$\omega^2 = 277.6$

$$\omega = \frac{2\pi N}{60} = 16.66$$

$N = \underline{159.1 \text{ rpm}}$

Also, range of speed = 159.1 – 147 = 12.1 rpm

Example 16.4 ***Each arm of a Porter governor is 250 mm long. The upper and lower arms are pivoted to links of 40 mm and 50 mm respectively from the axis of rotation. Each ball has a mass of 5 kg and the sleeve mass is 50 kg. The force of friction on the sleeve of the mechanism is 40 N. Determine the range of speed of the governor for extreme radii of rotation of 125 mm and 150 mm.***

Solution Refer Fig.16.7.

$m = 5$ kg $\quad AB = AE = 250$ mm

$M = 50$ kg $\quad BG = 50$ mm

$f = 40$ N $\quad EH = 40$ mm

(i) When $r = 125$ mm,

$$\sin\theta = \frac{125-40}{250} = 0.34 \qquad \theta = 19.88°$$

$$\tan\theta = \tan 19.88° = 0.362$$

$$\sin\beta = \frac{125-50}{250} = 0.3 \qquad \beta = 17.46°$$

$$\tan\beta = \tan 17.46° = 0.315$$

$$k = \frac{\tan\beta}{\tan\theta} = 0.87$$

As the radii decrease, the sleeve moves down and the force of friction f acts upwards.

$$mr\omega^2 = \tan\theta\left[mg + \frac{Mg-f}{2}(1+k)\right]$$

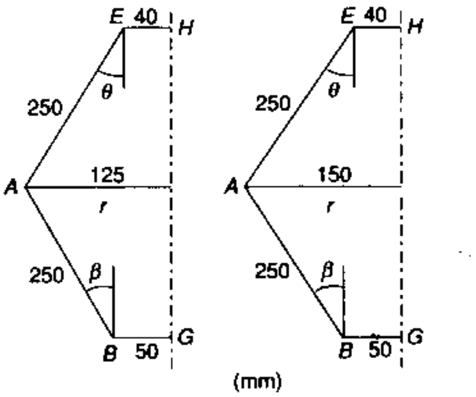


Fig. 16.7

$$5\times 0.125\omega^2 = 0.362\left[5\times 9.81 + \frac{50\times 9.81-40}{2}(1+0.87)\right]$$

$$\omega^2 = 272.4$$

$$\omega = \frac{2\pi N}{60} = 16.5$$

$$N_{min} = 157.6 \text{ rpm}$$

(ii) When $r = 150$ mm

$$\sin\theta = \frac{150-40}{250} = 0.44 \qquad \theta = 26.1°$$

$$\tan\theta = 0.49$$

$$\sin\beta = \frac{150-50}{250} = 0.4 \qquad \beta = 23.58°$$

$$\tan\beta = 0.436$$

$$k = \frac{0.436}{0.49} = 0.891$$

$$mr\omega^2 = \tan\theta\left[mg + \frac{Mg+f}{2}(1+k)\right]$$

(sleeve moves up)

$$5\times 0.15\omega^2 = 0.49\left[5\times 9.81 + \frac{50\times 9.81+40}{2}(1+0.891)\right]$$

$$\omega^2 = 359.8$$

$$\omega = \frac{2\pi N}{60} = 18.97$$

$$N_{max} = 181.1 \text{ rpm}$$

Range of speed = 157.6 rpm to 181.1 rpm = 23.5 rpm

Example 16.5 ***Each arm of a Porter governor is 200 mm long and is hinged at a distance of 40 mm from the axis of rotation. The mass of each ball is 1.5 kg and the sleeve is 25 kg. When the links are at 30° to the vertical, the sleeve begins to rise at 260 rpm. Assuming that the friction force is constant, find the maximum and the minimum speeds of rotation when the inclination of the arms to the vertical is 45°.***

Solution Refer Fig. 16.8.

$r = 200 \sin 30° + 40 = 140$ mm

$$h = \frac{r}{\tan 30°} = \frac{140}{\tan 30°} = 243 \text{ mm}$$

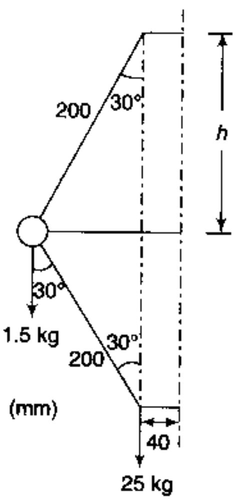


Fig. 16.8

At 30° angle, the sleeve begins to rise; therefore, the friction force is to act downwards.

$$\text{Thus,}\quad 260^2 = \frac{895}{h}\left(\frac{mg + (Mg + f)}{mg}\right)$$

$$= \frac{895}{0.243}\left(\frac{1.5 \times 9.81 + (25 \times 9.81 + f)}{1.5 \times 9.81}\right)$$

$$14.7 + 245.3 + f = 270$$

or $f = 10$ N

When the angle is 45°,

$r = 200 \sin 45° + 40 = 181.4$ mm

$$h = \frac{r}{\tan 45°} = \frac{181.4}{\tan 45°} = 181.4 \text{ mm}$$

$$N_1^2 = \frac{895}{0.1814}\left(\frac{1.5 \times 9.81 + (25 \times 9.81 + 10)}{1.5 \times 9.81}\right) = 90\,519$$

$N_1 = 300.9$ rpm

$$N_2^2 = \frac{895}{0.1814}\left(\frac{1.5 \times 9.81 + (25 \times 9.81 - 10)}{1.5 \times 9.81}\right) = 83\,812$$

-$N_2 = 289.5$ rpm

PROELL GOVERNOR

A Porter governor is known as a Proell governor if the two balls (masses) are fixed on the upward extensions of the lower links which are in the form of bent links *BAE* and *CDF* [Fig. 16.9(a)].

Considering the equilibrium of the link *BAE* which is under the action of [Fig. 16.9(b)]

- the weight of the ball, mg
- the centrifugal force, $mr'\omega^2$
- the tension in the link AO
- the horizontal reaction of the sleeve.
- the weight of sleeve and friction, $\frac{1}{2}(Mg \pm f)$

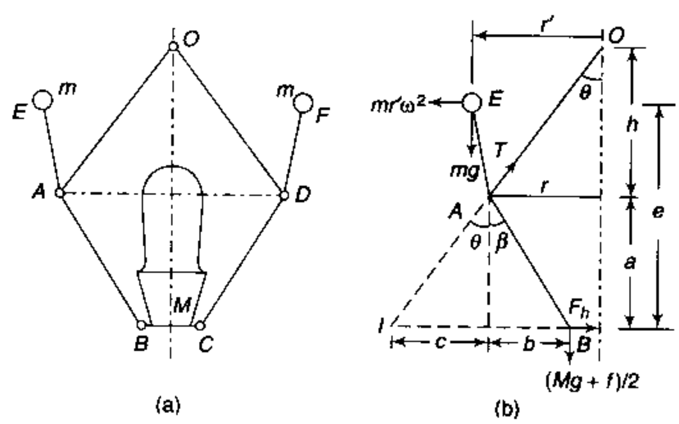


Fig. 16.9

As before, I is the instantaneous centre of the link BAE.

Taking moments about I,

$$mr'\omega^2 e = mg\,(c + r - r') + \frac{Mg \pm f}{2}(c + b)$$

where b, c, d and r are the dimensions as indicated in the diagram.

$$mr'\omega^2 = \frac{1}{e}\left[mg(c + r - r') + \frac{Mg \pm f}{2}(c + b)\right] \tag{16.4}$$

In the position when AE is vertical, i.e., neglecting its obliquity

$$mr'\omega^2 = \frac{1}{e}\left[mgc + \frac{Mg \pm f}{2}(c + b)\right]$$

$$= \frac{a}{e}\left[mg\frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a} + \frac{b}{a}\right)\right]$$

$$= \frac{a}{e}\left[mg\tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)\right]$$

$$= \frac{a}{e}\tan\theta\left[mg + \frac{Mg \pm f}{2}(1 + k)\right] \tag{16.5}$$

$$= \frac{a}{e}\frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1 + k)\right]$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{a}{e}\frac{g}{h}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right)$$

$$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right) \quad \text{(Taking } g = 9.81 \text{ m/s}^2\text{)}$$

- If $k = 1$, $$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{mg + (Mg \pm f)}{mg}\right)$$
- If $f = 0$, $$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{2m + M(1 + k)}{2m}\right)$$
- If $k = 1, f = 0$ $$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{m + M}{m}\right)$$

Example 16.6 ***Each arm of a Proell governor is 240 mm long and each rotating ball has a mass of 3 kg. The central load acting on the sleeve is 30 kg. The pivots of all the arms are 30 mm from the axis of rotation. The vertical height of the governor is 190 mm. The extension links of the lower arms are vertical and the governor speed is 180 rpm when the sleeve is in the mid-position. Determine the lengths of the extension links and the tension in the upper arms.***

Solution Refer Fig. 16.10.

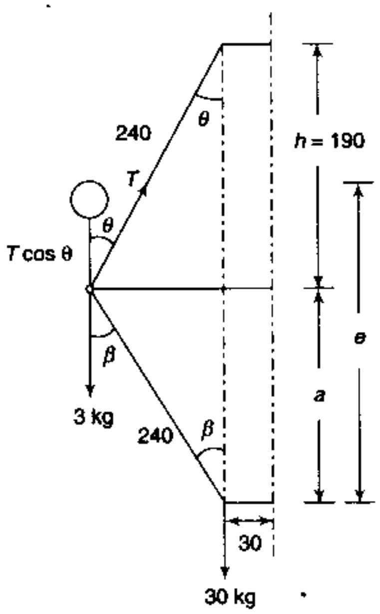


Fig. 16

$m = 3$ kg $\qquad\qquad M = 30$ kg

$h = 190$ mm

$$N^2 = \frac{895}{h} \cdot \frac{a}{e} \left(\frac{m + M}{m} \right)$$

$$180^2 = \frac{895}{0.19} \cdot \frac{0.19}{e} \left(\frac{3 + 30}{3} \right)$$

$e = 0.304$ m

Therefore, length of the extension links

$= e - a = 304 - 190 =$ 104 mm

Let T be the tension in the upper arms.

Considering the vertical components of the forces on the lower link.

$$T \cos \theta = mg + \frac{Mg}{2}$$

$$\cos \theta = \frac{0.19}{0.24} = 0.792$$

$$T \times 0.792 = 3 \times 9.81 + \frac{30 \times 9.81}{2}$$

$T =$ 223 N

Example 16.7 ***The mass of each ball of a Proell governor is 7.5 kg and the load on the sleeve is 80 kg. Each of the arms is 300 mm long. The upper arms are pivoted on the axis of rotation whereas the lower arms are pivoted to links of 40 mm from the axis of rotation. The extensions of the lower arms to which the balls are attached are 100 mm long and are parallel to the governor axis at the minimum radius. Determine the equilibrium speeds corresponding to extreme radii of 180 mm and 240 mm.***

Solution When AE is vertical, $r' = r =$ 180 mm[Fig. 16.11(a)].

$$mr\omega^2 = \frac{a}{e} \tan \theta \left[mg + \frac{Mg}{2}(1 + k) \right]$$

(friction neglected)

we have, $a = \sqrt{(300)^2 - (180 - 40)^2} = 265.3$ mm

$e = 265.3 + 100 = 365.3$ mm

$$\sin \theta = \frac{180}{300} = 0.6; \qquad \theta = 36.87°$$

$\tan \theta = 0.75$

$$\sin \beta = \frac{180 - 40}{300} = 0.467; \quad \beta = 27.82°$$

$\tan \beta = 0.528$

$$k = \frac{\tan \beta}{\tan \theta} = \frac{0.528}{0.75} = 0.704$$

$$7.5 \times 0.18 \times \omega^2 = \frac{0.2653}{0.3653} \times 0.75 \left[7.5 \times 9.81 + \frac{80 \times 9.81}{2}(1 + 0.704) \right]$$

$\omega^2 = 299.5$

$$\omega = \frac{2\pi N}{60} = 17.305$$

$N =$ 165.3 rpm

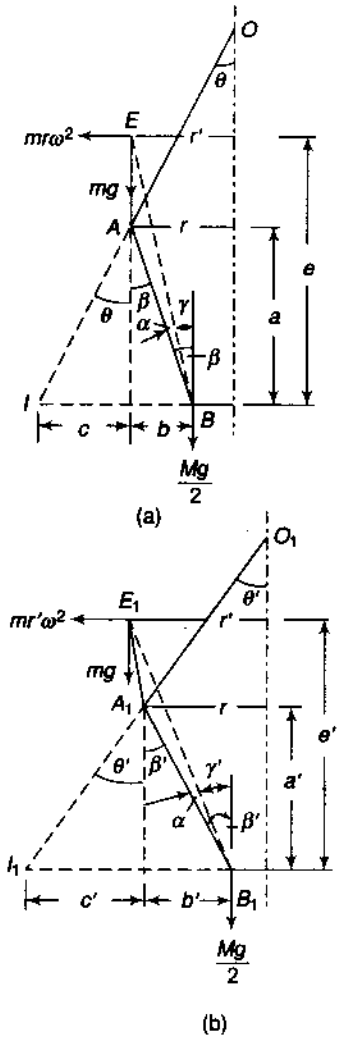


Fig. 16.11

$$BE = \sqrt{e^2 + b^2} = \sqrt{(365.3)^2 + (140)^2}$$

$$= 391.2 \text{ mm}$$

$$\cos\gamma = \frac{e}{BE} = \frac{365.3}{391.2} = 0.934 \quad \gamma = 20.97°$$

$$\sin\beta = \frac{b}{AB} = \frac{140}{300} = 0.467 \quad \beta = 27.82°$$

$$\alpha = 27.82° - 20.97° = 6.85°$$

$$\sin\beta' = \frac{b'}{A_1B_1} = \frac{240 - 40}{300} = 0.667 \quad \beta' = 41.81°$$

$$\therefore\ \gamma' = \beta' - \alpha = 41.81° - 6.85° = 34.96°$$

[Refer Fig.16.11(b)]

$$e' = B_1E_1 \cos\gamma' = BE\cos\gamma' = 391.2 \cos 34.96°$$
$$= 320.6 \text{ mm}$$

$$r' = B_1E_1 \sin\gamma' + 40 = 391.2 \sin 34.96° + 40$$
$$= 264.2 \text{ mm}$$

$$b' = 200 \text{ mm}$$

$$a' = A_1B_1 \cos\beta' = 300 \cos 41.81° = 223.6 \text{ mm}$$

$$\sin\theta' = \frac{240}{300} = 0.8 \qquad \theta' = 53.13°$$

$$c' = \alpha' \tan\theta' = 223.6 \tan 53.13° = 298.1 \text{ mm}$$

Taking moments about I,

$$mr'\omega^2 e' = mg(c' + r - r') + \frac{Mg}{2}(c' + b')$$

$$7.5 \times 0.2642 \times \omega^2 \times 0.3206$$
$$= 7.6 \times 9.81\ (0.2981 + 0.24 + 0.2642)$$
$$+ \frac{80 \times 9.81}{2}(0.2981 + 0.2)$$

$$\omega^2 = 339.4$$

$$\omega = \frac{2\pi N}{60} = 18.4$$

N = <u>175.9 rpm</u>

HARTNELL GOVERNOR

In this type of governor, the balls are controlled by a spring as shown in Fig. 16.12(a). Initially, the spring is fitted in compression so that a force is applied to the sleeve. Two bell-crank levers, each carrying a mass at one end and a roller at the other, are pivoted to a pair of arms which rotate with the spindle. The rollers fit into a groove in the sleeve.

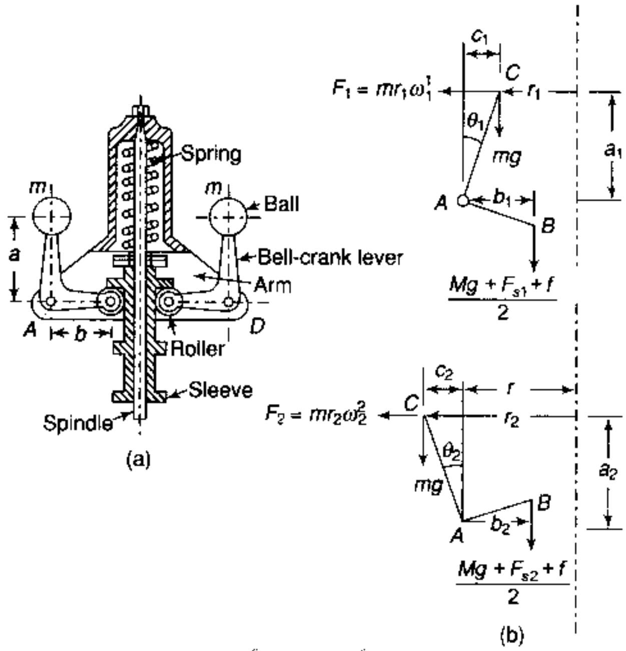
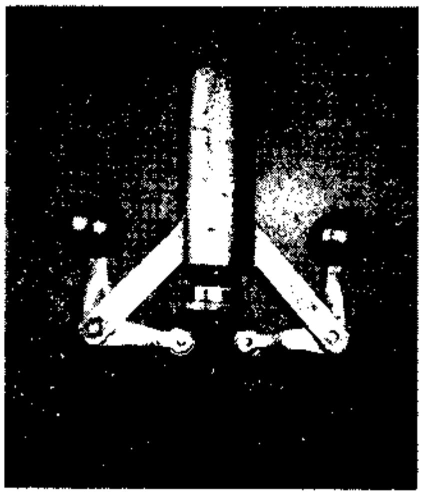


Fig. 16.12



A Hartnell governor

As the speed increases and the balls move away from the spindle axis, the bell-crank levers move on the pivot and lift the sleeve against the spring force. If the speed decreases, the sleeve moves downwards. The movement of the sleeve is communicated to the throttle of the engine. The spring force can be adjusted with the help of a screw cap.

Figure 16.12(b) shows the forces acting on the bell-crank lever in two positions (assuming that the sleeve moves up so that f is taken positive).

Let F = centrifugal force = $mr\omega^2$

F_s = spring force

Taking moments about the fulcrum A,

$$F_1 a_1 = \frac{1}{2}(Mg + F_{s1} + f)b_1 + mgc_1 \tag{16.6}$$

$$F_2 a_2 = \frac{1}{2}(Mg + F_{s2} + f)b_2 + mgc_2 \tag{16.6a}$$

In the working range of the governor, θ is usually small and so the obliquity effects of the arms of the bell-crank levers may be neglected. In that case,

$a_1 = a_2 = a$, $b_1 = b_2 = b$, $c_1 = c_2 = 0$

$$F_1 a = \frac{1}{2}(Mg + F_{s1} + f)b \tag{i}$$

and
$$F_2 a_2 = \frac{1}{2}(Mg + F_{s2} + f)b \tag{ii}$$

Subtracting (i) from (ii)

$$(F_2 - F_1)\,a = \frac{1}{2}(F_{s2} - F_{s1})b$$

or

$$F_{s2} - F_{s1} = \frac{2a}{b}(F_2 - F_1)$$

Let s = stiffness of the spring

h_1 = movement of the sleeve

$$F_{s2} - F_{s1} = h_1 s = \frac{2a}{b}(F_2 - F_1) \quad \text{or} \quad s = \frac{2}{h_1}.\frac{a}{b}.(F_2 - F_1)$$

But

$$h_1 = \theta.b = \frac{r_2 - r_1}{a}.b$$

$\therefore$

$$s = \frac{2}{r_2 - r_1}.\left(\frac{a}{b}\right)^2.(F_2 - F_1) = 2\left(\frac{a}{b}\right)^2\left(\frac{F_2 - F_1}{r_2 - r_1}\right) \qquad (16.7)$$

Example 16.8 *In a Hartnell governor, the extreme radii of rotation of the balls are 40 mm and 60 mm, and the corresponding speeds are 210 rpm and 230 rpm. The mass of each ball is 3 kg. The lengths of the ball and the sleeve arms are equal. Determine the initial compression and the constant of the central spring.*

Solution

$$\omega_1 = \frac{2\pi \times 210}{60} = 22 \text{ rad/s};$$

$$\omega_2 = \frac{2\pi \times 230}{60} = 23.04 \text{ rad/s}$$

$$F_1 = mr_1\omega_1^2 = 3 \times 0.04 \times 22^2 = 58.1 \text{ N}$$

and $F_2 = mr_2\omega_2^2 = 3 \times 0.06 \times 23.04^2 = 95.6$ N

Spring constant,

$$s = 2\left(\frac{a}{b}\right)^2\left(\frac{F_2 - F_1}{r_2 - r_1}\right)$$

$$= 2\,(1)^2\left(\frac{95.6 - 58.1}{60 - 40}\right) = \underline{3.75 \text{ N/mm}}$$

We have, $F_1 a = \frac{1}{2}(Mg + F_{s1} + f)b$ or $F_1 = \frac{F_{s1}}{2}$

$(M = 0, f = 0, a = b)$

or $F_{s1} = 2 \times 58.1 = 116.2$ N

Initial compression $= \frac{116.2}{3.75} = \underline{31 \text{ mm}}$

Example 16.9 *In a spring-loaded governor of the Hartnell type, the lengths of the horizontal and the vertical arms of the bell-crank lever are 40 mm and 80 mm respectively. The mass of each ball is 1.2 kg. The extreme radii of rotation of the balls are 70 mm and 105 mm. The distance of the fulcrum of each bell-crank lever is 75 mm from the axis of rotation of the governor. The minimum equilibrium speed is 420 rpm and the maximum equilibrium speed is 4% higher than this. Neglecting the obliquity of the arms, determine the*

(i) spring stiffness,

(ii) initial compression, and

(iii) equilibrium speed corresponding to radius of rotation of 95 mm.

Solution

$$\omega_1 = \frac{2\pi \times 420}{60} = 44 \text{ rad/s};$$

$$\omega_2 = 44 \times 1.04 = 45.76 \text{ rad/s}$$

$$F_1 = mr_1\omega_1^2 = 1.2 \times 0.07 \times 44^2 = 162.6 \text{ N}$$

and $F_2 = mr_2\omega_2^2 = 1.2 \times 0.105 \times 45.76^2 = 263.8$ N

(i) Spring constant,

$$s = 2\left(\frac{a}{b}\right)^2\left(\frac{F_2 - F_1}{r_2 - r_1}\right) = 2\left(\frac{80}{40}\right)^2\left(\frac{263.8 - 162.6}{105 - 70}\right)$$

$= \underline{23.14 \text{ N/mm}}$

(ii) We have, $F_1 a = \frac{1}{2}(Mg + F_{s1} + f)b$

or $F_1 = \frac{F_{s1}}{4}$ $\quad (M = 0, f = 0, a = 2b)$

or $F_{s1} = 4 \times 162.6 = 650.4$ N

Initial compression $= \frac{650.4}{23.14} = \underline{28.1 \text{ mm}}$

(iii) Let F_3 be the centrifugal force at $r_3 = 95$ mm,

Then $s = 2\left(\frac{a}{b}\right)^2\left(\frac{F_3 - F_1}{r_3 - r_1}\right)$

or $23.14 = 2(2)^2\left(\frac{F_3 - 162.6}{95 - 70}\right)$

or $F_3 = 162.6 + 72.3 = 234.9$ N

or $mr_3\omega_3^2 = 234.9$ or $1.2 \times 0.095 \times \omega^2 = 234.9$

or $\omega = 45.393$ rad/s or $\frac{2\pi N}{60} = 45.393$

or N = 433.5 rpm

* The distance of the fulcrum of each bell-crank lever from the axis of rotation of the governor (=75 mm) is superfluous data.

Example 16.10 *The arms of a Hartnell governor are of equal length. When the sleeve is in the mid-position, the masses rotate in a circle with a diameter of 150 mm (the arms are vertical in the mid-position). Neglecting friction, the equilibrium speed for this position is 360 rpm. Maximum variation of speed, taking friction into account, is to be 6% of the mid-position speed for a maximum sleeve movement of 30 mm. The sleeve mass is 5 kg and the friction at the sleeve is 35 N.*

Assuming that the power of the governor is sufficient to overcome the friction by 1% change of speed on each side of the mid-position, find (neglecting obliquity effect of arms), the

(i) mass of each rotating ball

(ii) spring stiffness

(iii) initial compression of the spring

Solution

$$\omega = \frac{2\pi \times 360}{60} = 37.7 \text{ rad/s}$$

(i) Considering the friction at the mid-position,

$$mr\omega_1^2 a = \frac{1}{2}(Mg + F_s + f)b$$

$$m \times \left(\frac{0.150}{2}\right) \times (37.7 \times 1.01)^2$$

$$= \frac{1}{2}(5 \times 9.81 + F_s + 35) \quad (a = b) \qquad \text{(i)}$$

and $mr\omega_2^2 a = \frac{1}{2}(Mg + F_s - f)b$

$$m \times \left(\frac{0.150}{2}\right) \times (37.7 \times 0.99)^2$$

$$= \frac{1}{2}[5 \times 9.81 + F_s - 35] \qquad \text{(ii)}$$

Subtracting (ii) from (i)

$m \times 0.075 \times (37.7)^2\ [(1.01)^2 - (0.99)^2]$

$= \frac{1}{2} \times (35 + 35)$

or $m = \underline{8.21 \text{ kg}}$

(ii) In the extreme positions,

$$mr_2\omega_2^2 a = \frac{1}{2}(Mg + F_{s2} + f)b$$

$$8.21 \times \left(0.075 + \frac{0.03}{2}\right) \times (37.7 \times 1.06)^2$$

$$= \frac{1}{2}(5 \times 9.81 + F_{s2} + 35) \quad (a = b)$$

$F_{s2} = 2275.8$ N

$$mr_1\omega_1^2 a = \frac{1}{2}(Mg + F_{s1} - f)b$$

$$8.21 \times \left(0.075 - \frac{0.03}{2}\right) \times (37.7 \times 0.94)^2$$

$$= \frac{1}{2}(5 \times 9.81 + F_{s1} - 35)$$

$F_{s1} = 1223.2$ N

$h_1 s = F_{s2} - F_{s1}$

$0.03 \times s = 2275.8 - 1223.2$

$s = 35\,088$ N/m or $\underline{35.088 \text{ N/mm}}$

(iii) Initial compression $= \frac{F_{s1}}{s} = \frac{1223.2}{35.088}$ $= \underline{34.86 \text{ mm}}$

Example 16.11 *In a spring-loaded Hartnell type of governor, the mass of each ball is 4 kg and the lift of the sleeve is 40 mm. The governor begins to float at 200 rpm when the radius of the ball path is 90 mm. The mean working speed of the governor is 16 times the range of speed when friction is neglected. The lengths of the ball and roller arms of the bell-crank lever are 100 mm and 80 mm respectively. The pivot centre and the axis of governor are 115 mm apart. Determine the initial compression of the spring, taking into account the obliquity of arms.*

Assuming the friction at the sleeve to be equivalent to a force of 15 N, determine the total alteration in speed before the sleeve begins to move from the mid-position.

Solution Refer Fig. 16.13.

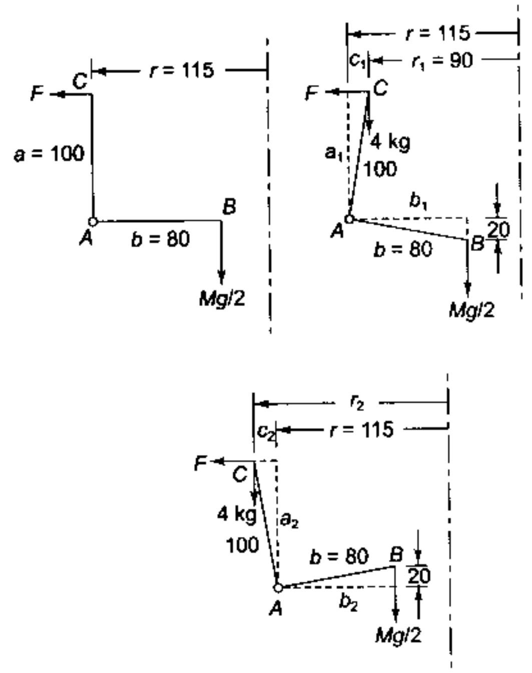


Fig. 16.13

$m = 4$ kg $\qquad N_1 = 200$ rpm

$h_1 = 40$ mm $\qquad r_1 = 90$ mm

$a = 100$ mm $\qquad r = 115$ mm

$b = 80$ mm

Mean speed, $N = \dfrac{N_1 + N_2}{2}$

As $\quad N = 16\,(N_2 - N_1)$

$\therefore \quad \dfrac{N_1 + N_2}{2} = 16\,(N_2 - N_1)$

or $\quad \dfrac{200 + N_2}{2} = 16\,(N_2 - 200)$

$N_2 = 212.9$ rpm

Angle turned by bell-crank lever between two extreme positions

$$= \frac{\text{Lift}\,(h_1)}{b} = \frac{c_1 + c_2}{a}$$

or $\quad c_1 + c_2 = h_1\dfrac{a}{b} = 40 \times \dfrac{100}{80} = 50$ mm

But $\quad c_1 = r - r_1 = 115 - 90 = 25$ mm

$c_2 = 50 - 25 = 25$ mm

$r_2 = r + c_2 = 115 + 25 = 140$ mm

$$b_1 = b_2 = \sqrt{b^2 - (h/2)^2}$$

$$= \sqrt{(80)^2 - (20)^2} = 77.46 \text{ mm}$$

$$a_1 = a_2 = \sqrt{(100)^2 - (25)^2} = 96.82 \text{ mm}$$

$$\omega_1 = \frac{2\pi \times 200}{600} = 20.94 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi \times 212.9}{60} = 22.29 \text{ rad/s}$$

In the extreme positions,

$$m r_1 \omega_1^2 a_1 = \frac{1}{2} F_{S1} b_1 + mg\, c_1 \qquad (M = 0, f = 0)$$

$$4 \times 0.09 \times (20.94)^2 \times 0.096\,82 = \frac{1}{2}$$

$$F_{s1} \times 0.077\,46 + 4 \times 9.81 \times 0.025$$

$F_{s1} = 392.5$ N

$$m r_2 \omega_2^2 a_2 = \frac{1}{2} F_{s2} b_2 + mg\, c_2$$

$$4 \times 0.14 \times (22.29)^2 \times 0.096\,82 = \frac{1}{2}$$

$$F_{s2} \times 0.077\,46 + 4 \times 9.81 \times 0.025$$

$F_{s2} = 698$ N

$h_1 s = F_{s2} - F_{s1}$

$40 \times s = 698 - 392.5$

$s = 7.64$ N/mm

Initial compression $= \frac{F_{s1}}{s} = \frac{392.5}{7.64} = \underline{51.37 \text{ mm}}$

F_s at mid-position $= F_{s1} + 20\,s$

$= 392.5 + 7.64 \times 20 = 543.3$ N

Mean speed $= \frac{N_1 + N_2}{2}$

$= \frac{212.9 - 200}{2} = 206.45$ rpm

At the mid-position, taking friction into account,

$mr\omega^2 a = \frac{1}{2}(F_s + f)b$

$4 \times 0.115 \times \omega^2 \times 0.1 = \frac{1}{2}(545.3 + 15) \times 0.08$

$\omega_1^2 = 487.2$

$\omega_1 = \frac{2\pi N_1}{60} = 22.07$

$N_1 = 210.8$ rpm

Also $mr\omega_2^2 a = \frac{1}{2}(F_S - f)b$

$4 \times 0.115 \times \omega_2^2 \times 0.1 = \frac{1}{2}(545.3 - 15) \times 0.08$

$\omega_2^2 = 461.13$

$\omega_2 = \frac{2\pi N}{60} = 21.47$

$N_2 = 205.1$ rpm

Alteration at speed $= 210.8 - 205.1 = \underline{5.7 \text{ rpm}}$

16.6 HARTUNG GOVERNOR

A Hartung type of governor is shown in Fig. 16.14. It is a spring-controlled governor in which the vertical arms of the bell-crank lever are fitted with spring balls. The springs compress against the frame of the governor while the rollers at the horizontal arm press against the sleeve.

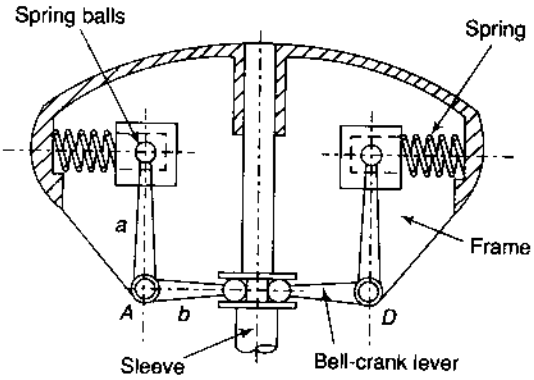


Fig. 16.14

Let F = centrifugal force

m = mass of each ball

S = spring force

s = stiffness of the spring

M = mass of sleeve

r = radial distance of the masses

ω = angular velocity of the balls at radius r

r_o = radius at which the spring force is zero

a = length of vertical arm of bell-crank lever

b = length of horizontal arm of bell-crank lever

Neglecting the obliquity of the arms and taking moments about the fulcrum A,

$$F.a = S.a + \frac{Mg}{2}.b$$

or

$$mr\omega^2.a = s(r - r_o).a + \frac{Mg}{2}.b$$

Example 16.12 *In a spring-controlled Hartung type of governor, the length of the ball arm is 84 mm and the sleeve arm is 126 mm. When in the mid-position, each spring is compressed by 60 mm and the radius of rotation of the mass centres is 160 mm. The mass of the sleeve is 18 kg and each ball is 4 kg. The spring stiffness is 12 kN/m of compression and total lift of the sleeve is 24 mm. Determine the ratio of the range of speed to the mean speed of the governor. Also find the speed in the mid-position. Neglect the moment due to the revolving masses when the arms are inclined.*

Solution

$m = 4$ kg	$r = 160$ mm
$M = 18$ kg	$s = 12$ kN/m
$a = 84$ mm	$r_o = 160\text{-}60 = 100$ mm
$b = 126$ mm	$h = 24$ mm

In the mid-position [Fig. 16.15(a)],

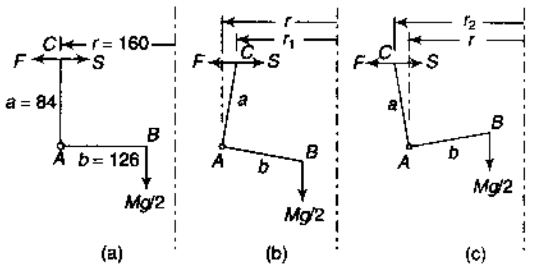


Fig. 16.15

$$mr\omega^2.a = s(r - r_o).a + \frac{Mg}{2}.b$$

$$4 \times 0.16\ \omega^2 \times 0.084 = 12\ 000 \times 0.06. \times 0.084 + \frac{18 \times 9.81}{2} \times 0.126$$

$$= 60.48 + 11.125$$

$$\omega^2 = 1332$$

$$\omega = 36.5 \text{ rad/s}$$

$$= \frac{36.5 \times 60}{2\pi} \text{ or } 348.5 \text{ rpm}$$

Thus, mean speed = 348.5 rpm

For the minimum speed, from the Fig. 16.15.(b), (neglecting obliquity of arms)

$$\frac{r - r_1}{a} = \frac{h}{b}$$

or $r_1 = r - h.\frac{a}{b} = 0.16 - \frac{0.24}{2}.\frac{0.084}{0.126} = 0.152$ mm

$$4 \times 0.152\omega^2 \times 0.084 = 12\ 000 \times (0.152\text{–}0.1) \times 0.084 + \frac{18 \times 9.81}{2} \times 0.126$$

$$= 52.416 + 11.125$$

$$\omega^2 = 1244$$

$$\omega = 35.27 \text{ rad/s}$$

$$= \frac{35.27 \times 60}{2\pi} \text{ or } 336.8 \text{ rpm}$$

Thus, minimum speed = 336.8 rpm

For the maximum speed, from the Fig. 16.15 (c), (neglecting obliquity of arms)

$$\frac{r_2 - r}{a} = \frac{h}{b}$$

or $r_2 = r + h.\frac{a}{b} = 0.16 + \frac{0.24}{2}.\frac{0.084}{0.126} = 0.168$ mm

$$4 \times 0.168\ \omega^2 \times 0.084 = 12\ 000 \times (0.168 - 0.1) \times 0.084 + \frac{18 \times 9.81}{2} \times 0.126$$

$$= 68.544 + 11.125$$

$$\omega^2 = 1411.4$$

$$\omega = 37.57 \text{ rad/s}$$

$$= \frac{37.57 \times 60}{2\pi} \text{ or } 358.75 \text{ rpm}$$

Thus, mean speed = 358.75 rpm

Range of speed = 358.75 – 336.8 = 21.95 rpm

Ratio of range of speed to mean speed

$$= \frac{21.95}{348.5} = 0.063$$

16.7 WILSON–HARTNELL GOVERNOR (RADIAL-SPRING GOVERNOR)

A Wilson–Hartnell governor is a spring-loaded type of governor. In this, two bell-crank levers are pivoted at the ends of two arms which rotate with the spindle [Fig. 16.16(a)]. The vertical arms of the bell-crank levers support the two balls at their ends while the horizontal arms carry two rollers at their ends. The two balls are connected by two main springs arranged symmetrically on either side of the sleeve. While rotating, when the ball radius increases with the increase in speed, the springs exert an inward pull F_s on the balls and the rollers press against the sleeve which is raised, closing the throttle value.

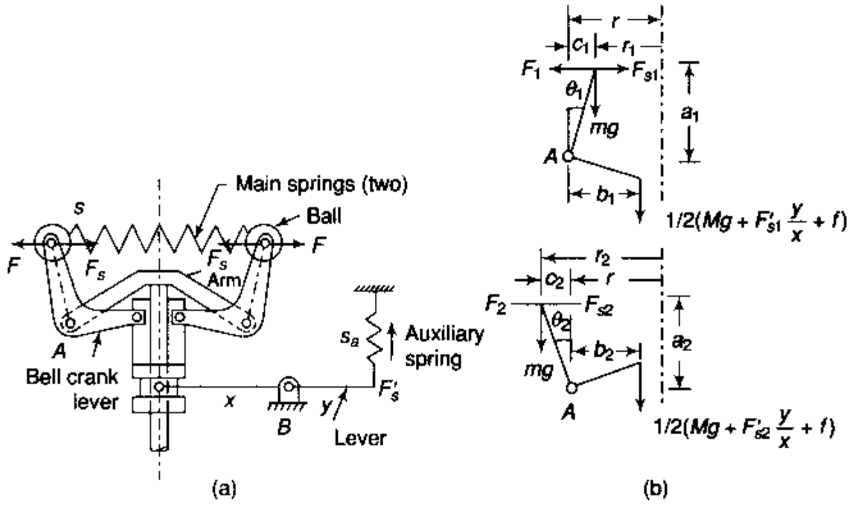


Fig. 16.16

Usually, the main springs are not adjustable and, for this reason, an adjustable auxiliary spring is provided. It is attached to one end of a lever, the other end of which fits into a groove in the sleeve. The lever is pivoted at a fulcrum B. The auxiliary spring tends to keep the sleeve down so that it assists each main spring, i.e., main and the auxiliary springs are in tension simultaneously.

Let s = stiffness of each of the main springs

S_a = stiffness of the auxiliary spring

F'_s = force applied by the auxiliary spring

Assuming that the sleeve moves up, take moments about the fulcrum A in two positions [Fig. 16.16(b)],

$$F_1 a_1 - F_{s1} a_1 = \frac{1}{2}\left(Mg + F'_{s1}\frac{y}{x} + f \right) b_1 + mg\, c_1 \tag{16.8}$$

$$F_2 a_2 - F_{s2} a_2 = \frac{1}{2}\left(Mg + F'_{s2}\frac{y}{x} + f \right) b_2 + mg\, c_2 \tag{16.8a}$$

If obliquity effects are neglected,

$$a_1 = a_2 = a, \quad b_1 = b_2 = b \quad \text{and} \quad c_1 = c_2 = 0$$

$$(F_1 - F_{s1})\, a = \frac{1}{2}\left(Mg + F'_{s1}\frac{y}{x} + f \right) b \tag{i}$$

$$(F_2 - F_{s2})\, a = \frac{1}{2}\left(Mg + F'_{s2}\frac{y}{x} + f \right) b \tag{ii}$$

Subtracting (i) form (ii),

$$a\,(F_2 - F_1) - a\,(F_{s2} - F_{s1}) = (F'_{s2} - F'_{s1})\,\frac{yb}{2x} \tag{iii}$$

The main spring consists of two springs. Therefore, the force exerted is given by,

$F_{s2} - F_{s1}$ = 2 × Force exerted by each spring

= 2 × Stiffness of each spring × Elongation of each spring

$= 2 \times s \times 2 \times (r_2 - r_1)$
$= 4s\,(r_2 - r_1)$

Let h_1 = movement of the sleeve
and h_2 = deflection of the auxiliary spring
Then

$$F'_{s2} - F'_{s1} = h_{2\,Sa}$$
$$= \left(h_1 \frac{y}{x}\right) S_a$$
$$= (r_2 - r_1)\frac{b}{a}\frac{y}{x} S_a$$

Then (iii) becomes,

$$a(F_2 - F_1) - 4as(r_2 - r_1) = (r_2 - r_1)\frac{b}{a}\frac{y}{x} S_a \frac{yb}{2x}$$

or
$$(F_2 - F_1) - 4s(r_2 - r_1) + (r_2 - r_1)\frac{S_a}{2} S_a \left(\frac{b}{a}\frac{y}{x}\right)^2$$

or
$$\frac{F_2 - F_1}{r_2 - r_1} = 4s + \frac{S_a}{2}\left(\frac{b}{a}\frac{y}{x}\right)^2 \qquad (16.9)$$

To find the stiffness of the main springs while using this equation, the stiffness of the auxiliary spring may be fixed first.

Example 16.13 ***In a Wilson–Hartnell type of governor, the mass of each ball is 5 kg. The lengths of the ball arm and the sleeve arm of each bell-crank lever are 100 mm and 80 mm respectively. The stiffness of each of the two springs attached directly to the balls is 0.4 N/mm. The lever for the auxiliary spring is pivoted at its midpoint. When the radius of rotation is 100 mm, the equilibrium speed is 200 rpm. If the sleeve is lifted by 8 mm for an increase of speed of 6%, find the required stiffness of the auxiliary spring.***

Solution

$m = 5$ kg	$s = 0.4$ N/mm = 400 N/m
$r_1 = 100$ mm	$a = 100$ mm
$N_1 = 200$ rpm	$b = 80$ mm
	$y/x = 1$

We have,

$$\frac{F_2 - F_1}{r_2 - r_1} = 4s + \frac{S_a}{2}\left(\frac{b}{a} \times \frac{y}{x}\right)^2$$

When $r_1 = 100$ mm, $N_1 = 200$ rpm.

$$\omega_1 = \frac{200 \times 2\pi}{60} = 20.94 \text{ rad/s}$$

$F_1 = m r_1 \omega_1^2 = 5 \times 0.1 \times (20.94)^2 = 219.2$ N

For 6% rise of speed,

$\omega^2 = 20.94 \times 1.06 = 22.2$ rad/s

For sleeve rise of 8 mm,

Increase in ball radius $= 8 \times \frac{100}{80} = 10$ mm

$r_2 = 100 + 10 = 110$ mm

$F_2 = m r_2\, \omega_2^2 = 5 \times 0.11 \times (22.2)^2 = 271.1$ N

$$\frac{271.1 - 219.2}{0.11 - 0.1} = 4 \times 400 + \frac{S_a}{2}\left(\frac{0.08}{0.1} \times 1\right)^2$$

(Refer Eq. (16.9)]

$S_a = 11\,219$ N/m or 11.219 N/mm

16.8 PICKERING GOVERNOR

A Pickering governor consists of three leaf springs which are arranged at equal angular intervals around the governor spindle (Fig. 16.17), only one leaf spring is shown in the figure. The upper end of each spring is fixed by a screw to a hexagonal nut attached to the spindle. The lower end is fastened to the sleeve which can move up and down the governor spindle. Each spring has a fly mass m attached at its centre. As the spindle rotates, a centrifugal force is exerted on the leaf spring at the centre which causes it to deflect. This deflection makes the sleeve move up. A stop is also provided to limit the movement of the sleeve.

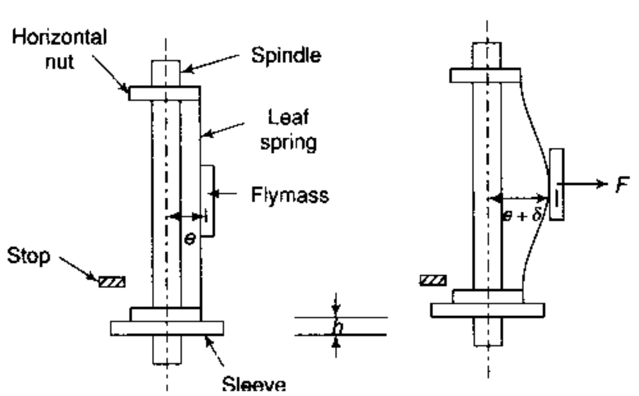


Fig. 16.17

Let m = mass fixed to each spring

e = distance between spindle axis and centre of mass when the governor is at rest

ω = Angular speed of the sleeve

δ = deflection of the centre of the leaf spring for spindle speed ω

Centrifugal force, $F = m(e + \delta)\,\omega^2$

To find δ, the leaf spring is treated as a beam of uniform cross section fixed at both ends and carrying a load at the centre.

$$\delta = \frac{Fl^3}{192EI} = \frac{m(e+\delta)\omega^2 l^3}{192EI}$$

where

E = modulus of elasticity of the spring material

I = moment of inertia of the cross-section of the spring about neutral axis $= \frac{bt^3}{12}$, b and t being the width and the thickness of the leaf spring.

An empirical relation to between the deflection δ and the lift h of the sleeve may also be used as follows:

$$h = 2.4\frac{\delta^2}{l}$$

A Pickering governor is used in gramophones to adjust the speed of the turn table.

Example 16.14 ***Each spring of a Pickering governor of a gramophone is 6 mm wide and 0.12 mm thick with a length of 48 mm. A mass of 25 g is attached to each leaf spring at the centre. The distance between the spindle axis and the centre of mass when the governor is at rest is 8 mm. The ratio of the governor speed to the turn table speed is 10. Determine the speed of the turn table for a sleeve lift of 0.6 mm. Take E = 200 GN/m².***

Solution

$m = 0.025$ kg $\qquad b = 0.006$ m

$e = 0.008$ m $\quad t = 0.000\ 12$ mm
$h = 0.6$ mm $\quad l = 48$ mm
$E = 200 \times 10^9$ N/m^2

$$I = \frac{bt^3}{12} = \frac{0.006 \times 0.000\ 12^3}{12} = 0.864 \times 10^{-15} \text{m}^4$$

Lift of the sleeve, $h = 2.4 \dfrac{\delta^2}{l}$

or $\quad 0.6 = 2.4 \dfrac{\delta^2}{48}$

or $\quad \delta = 3.464$ mm $= 0.003\ 464$ m

Now,

$$\delta = \frac{Fl^3}{192EI} = \frac{m(e+\delta)\omega^2 l^3}{192EI}$$

$$0.003464 = \frac{0.025(0.008 + 0.003\,464)\omega^2 \times 0.048^3}{192 \times 200 \times 10^9 \times 0.864 \times 10^{-15}}$$

$\omega^2 = 3626$

or $\quad \omega = 60.22$ rad/s

$$= \frac{60.22 \times 60}{2\pi} = 575 \text{ rpm}$$

Therefore, speed of the turn table $= \dfrac{575}{10} = 57.5$ rpm

16.9 SPRING-CONTROLLED GRAVITY GOVERNOR

In a spring-controlled gravity governor, two bell-crank levers are pivoted on the moving sleeve [Fig.16.18(a)]. The rollers at the ends of the horizontal arms of the levers press against a cap fixed to the governor shaft. Thus, the motion of the pivots will be vertically upwards whereas the rollers will be able to move horizontally over the cap. As the speed increases, the balls move away, the pivots are raised and the spring is compressed between the sleeve and the cap.

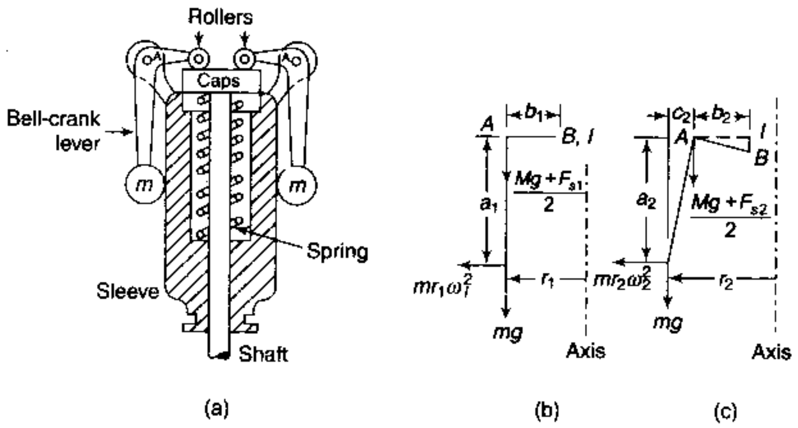


Fig. 16.18

Example 16.15 *In a spring-controlled gravity governor, the mass of each ball is 1.6 kg. The distance of fulcrum from the axis of rotation is 60 mm. The bell-crank lever has a 120 mm long vertical arm and a 50-mm long horizontal arm. The mass of the sleeve is 6.5 kg. The sleeve begins to rise at 200 rpm and the rise of sleeve for 5% increase is 9 mm. Determine the initial thrust in the spring and its stiffness.*

Solution

$m = 1.6$ kg $\quad N_1 = 200$ rpm
$M = 6.5$ kg $\quad a = a_1 = 200$ mm
$r_1 = 60$ mm $\quad b = b_1 = 50$ mm

$$\omega_1 = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s}$$

(i) For initial (neutral) position, taking moments about B, the I-centre [Fig.16.18(b)],

$$mr_1\omega_1^2 a_1 = mg\, b_1 + \frac{Mg + F_{s1}}{2} b_1$$

where F_{s1} is the spring load on the sleeve. The total sleeve load $(Mg + F_{s1})$ acts on the levers through the fulcrums $A - A$.

Thus,

$$\left[1.60 \times 0.06 \times (20.94)^2\right] \times 0.12$$
$$= \left[1.6 \times 9.891 + \frac{6.5 \times 9.81 + F_{S1}}{2}\right] \times 0.05$$

$\therefore$ initial thrust, $F_{s1} = \underline{107\text{ N}}$

(ii) When the sleeve rises through 9 mm, the radius is increased by c_2 [Fig.16.18(c)],

$$c_2 = 9 \times \frac{120}{50} = 21.6\text{ mm}$$

or $r_2 = 60 + 21.6 = 81.6$ mm

$\omega_2 = 20.94 \times 1.05 = 21.99$ rad/s

$$a_2 = \sqrt{a^2 - c_2^2} = \sqrt{(120)^2 - (21.6)^2}$$
$$= 118\text{ mm}$$

$$b_2 = \sqrt{(50)^2 - (9)^2} = 49.2\text{ mm}$$

Since the point A can move vertically and the point B horizontally, the I-centre of the lever BAC will be at I. Taking moments about this point,

$$(mr_2\omega_2^2)a_2 = mg\,(b_2 + c_2) + \frac{Mg + F_{s2}}{2} b_2$$

$$1.6 \times 0.0816 \times (21.99)^2 \times 0.118 = 1.6 \times 9.81$$
$$(0.0492 + 0.0216) + \frac{6.5 \times 9.81 + F_{s2}}{2} \times 0.0492$$

$F_{s2} = 193.8$ N

Stiffness of spring

$$= \frac{F_{s2} - F_{s1}}{h_2} = \frac{193.8 - 107}{9} = \underline{9.6\text{ N/mm}}$$

16.10 INERTIA GOVERNOR

As described earlier, an inertia governor is based on the principle of inertia of matter and is operated by the acceleration or deceleration of the rotating masses in addition to centrifugal forces.

In this type of governor, a mass m, having its centre at G, is fixed to an arm QG which is pivoted to a rotating disc on the engine shaft at Q. The points Q, G and the centre of rotation O are not to be collinear (Fig.16.19). The arm QG is connected to an eccentric that operates the fuel supply valve. Whenever the arm moves relatively to the disc, it shifts the position of the eccentric which changes the fuel supply.

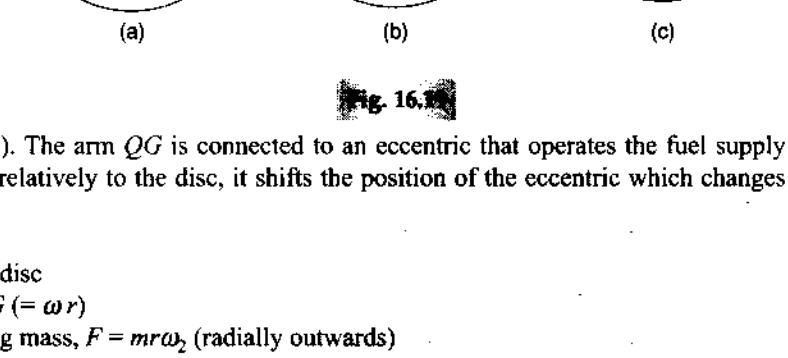


Fig. 16.19

Let r = radial distance OG

ω = angular velocity of the disc

v = tangential velocity of G $(= \omega r)$

Centrifugal force of the rotating mass, $F = mr\omega_2$ (radially outwards)

If the engine shaft is accelerated due to increase in speed, the ball mass does not get accelerated at the same amount on account of its inertia, the inertia force being equal to

$$F_i = mf = m\frac{dv}{dt}$$

Moment of F about $Q = mr\omega^2 x$ (counter-clockwise)

Moment of F_i about $Q = m\frac{dv}{dt} y$ (counter-clockwise)

Thus, it is seen that the moments due to the two forces add together to make the governor action rapid. Note that, as the mass moves outwards, the arm rotates in a direction opposite to that of the rotation of the shaft. In case the arm is arranged on the disc in a manner shown in Fig.16.19(b), the two moments due of F and F_i act in the opposite directions to make the governor action sluggish. This arrangement is, therefore, avoided.

It is also possible to use ball masses fixed to the arm as shown in Fig.16.19(c). The arm is pivoted at its midpoint C. A change in the angular speed of the disc makes the ball masses to have an angular movement about C. If I_c is the moment of inertia of the arm and the masses about an axis through C, then

torque on the arm = $I_c \, d\omega/dt$

Note that in an inertia governor, when the acceleration (or deceleration) is very small or the change in velocity is very slow, the additional inertia force is practically zero and an inertia governor in effect, becomes a centrifugal governor.

Example 16.16 ***Figure 16.20 shows the arrangement of an inertia governor. The disc rotates about the centre O. Two arms of negligible masses are pivoted at A and B which are 80 mm apart. Each arm has a mass of 300 g attached at the other end as shown in the figure. The distance of the centre of each mass from the respective pivot is 60 mm. Points C and D on the arms at 25 mm from the pivots are connected by a spring. It is ensured by a linkage that the angles θ_1 and θ_2 remain equal. The spring stiffness is 4 N/mm. Determine the***

(i) tension in the spring when each angle, i.e., θ_1 and θ_2 is 30° and the speed is 210 rpm

(ii) speed of rotation when rotating in the counter-clockwise direction the governor accelerates at a rate of 40 rad/s², each angle becomes 45°

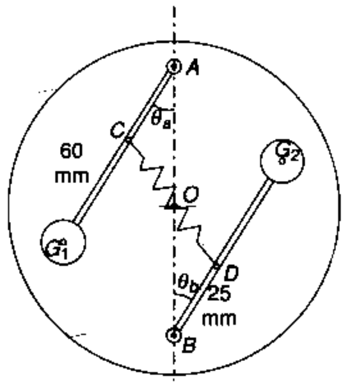


Fig. 16.20

Solution As the arrangement is symmetrical, forces on only one half may be considered for the equilibrium purposes.

(i) Draw the configuration to scale as shown in Fig. 16.21(a) for each angle of 30°.

$$\omega = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

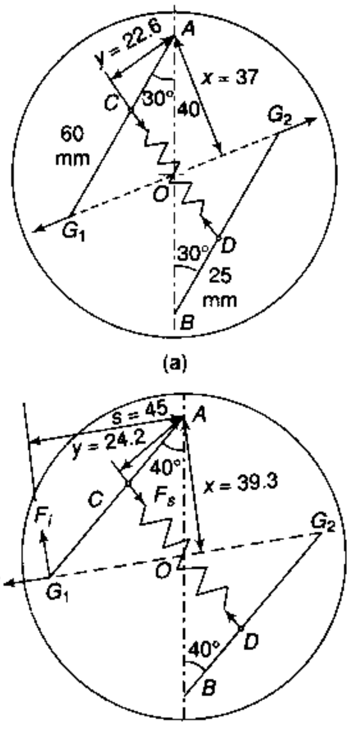


Fig. 16.21

On measurement, the perpendicular distance of the centrifugal force, $x = 37$ mm

And the perpendicular distance of the spring force, $y = 22.6$ mm

$OG_1 = 32.3$ mm

Centrifugal force $F = mr\omega^2$

$= 0.3 \times 0.0323 \times 22^2 = 4.69$ N

Taking moments about the pivot A,

$F \times x = F_s \times y$

or $4.69 \times 37 = F_s \times 22.6$

or $F_s = 7.68$ N

(ii) Draw the configuration to scale as shown in Fig. 16.21b for each angle of 40°.

On measurement, the perpendicular distance of the centrifugal force, $x = 39.3$ mm and the perpendicular distance of the spring force, $y = 24.2$ mm

⊥ distance of F_i, $s = 45$ mm

$OG_1 = 39$ mm

Elongation of spring = 53.05 – 45.15 = 7.9 mm

Centrifugal force $F = mr\omega^2 = 0.3 \times 0.039 \times \omega^2$

$= 0.0117\ \omega^2$ N

Inertia force, F_i = mass × tangential acceleration

$= 0.3 \times (40 \times 0.39)$

$= 4.68$ N

Spring force = Initial force + Stiffness × Elongation

$= 7.68 + 4 \times 7.9$

$= 39.28$ N

Taking moments about the pivot A,

$F \times x + F_i \times s = F_s \times y$

or $0.0117\ \omega_2 \times 39.3 + 4.68 \times 45 = 39.28 \times 24.2$

or $\omega_2 = 1609.3$

$\omega = 40.1$ rad/s

$\frac{2\pi N}{60} = 40.1$

$N = 383$ rpm

16.11 SENSITIVENESS OF A GOVERNOR

A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of sensitivity.

As a governor is used to limit the change of speed of the engine between minimum to full-load conditions, the sensitiveness of a governor is also defined as the ratio of the difference between the maximum and the minimum speeds (range of speed) to the mean equilibrium speed. Thus,

$$\text{Sensitiveness} = \frac{\text{range of speed}}{\text{mean speed}}$$

$$= \frac{N_2 - N_1}{N}$$

$$= \frac{2(N_2 - N_1)}{N_1 + N_2} \quad (16.10)$$

When N = mean speed

N_1 = minimum speed corresponding to full load conditions

N_2 = maximum speed corresponding to no-load conditions

16.12 HUNTING

Sensitiveness of a governor is a desirable quality. However, if a governor is too sensitive, it may fluctuate continuously, because when the load on the engine falls, the sleeve rises rapidly to a maximum position. This shuts off the fuel supply to the extent to affect a sudden fall in the speed. As the speed falls to below the mean value, the sleeve again moves rapidly and falls to a minimum position to increase the fuel supply. The speed subsequently rises and becomes more than the average with the result that the sleeve again rises to reduce the fuel supply. This process continues and is known as *hunting*.

16.13 ISOCHRONISM

A governor with a range of speed zero is known as an *isochronous governor*. This means that for all positions of the sleeve or the balls, the governor has the same equilibrium speed. Any change of speed results in moving the balls and the sleeve to their extreme positions. However, an isochronous governor is not practical due to friction at the sleeve.

For a Porter governor, with all arms equal in length and intersecting on the axis (neglecting friction),

$$h_1 = \frac{g}{\omega_1^2}\left(1+\frac{M}{m}\right) \text{ and } h_2 = \frac{g}{\omega_2^2}\left(1+\frac{M}{m}\right)$$

For isochronism, $\omega_1 = \omega_2$ and thus $h_1 = h_2$. However, from the configuration of a Porter governor, it can be judged that it is impossible to have two positions of the balls at the same speed. Thus, a pendulum type of governor cannot possibly be isochronous.

In the case of a Hartnell governor (neglecting friction),

At ω_1,

$$m r_1 \omega_1^2 a = \frac{1}{2}(Mg + F_{s1})b$$

At ω_2,

$$m r_2 \omega_2^2 a = \frac{1}{2}(Mg + F_{s2})b$$

For isochronism, $\omega_1 = \omega_2$.

$$\therefore \quad \frac{Mg + F_{s1}}{Mg + F_{s2}} = \frac{r_1}{r_2} \tag{16.11}$$

which is the required condition of iscohronism.

16.14 STABILITY

A governor is said to be stable if it brings the speed of the engine to the required value and there is not much hunting. The ball masses occupy a definite position for each speed of the engine within the working range.

Obviously, the stability and the sensitivity are two opposite characteristics.

16.15 EFFORT OF A GOVERNOR

The effort of the governor is the mean force acting on the sleeve to raise or lower it for a given change of speed. At constant speed, the governor is in equilibrium and the resultant force acting on the sleeve is zero. However, when the speed of the governor increases or decreases, a force is exerted on the sleeve which tends to move it. When the sleeve occupies a new steady position, the resultant force acting on it again becomes zero.

If the force acting at the sleeve changes gradually from zero (when the governor is in the equilibrium position) to a value E for an increased speed of the governor, the mean force or the effort is $E/2$.

For a Porter governor, the height is given by

$$h = \frac{g}{\omega^2} + \frac{Mg(1+k)}{2m\omega^2} = \frac{2mg + Mg(1+k)}{2m\omega^2} \quad \text{(i)}$$

Let ω be increased by c times ω where c is a factor and E be the force applied on the sleeve to prevent it from moving. Thus, the force on the sleeve is increased to $(Mg + E)$. Then

$$h = \frac{2mg + (Mg+E)(1+k)}{2m(1+c)^2\omega^2} \quad \text{(ii)}$$

Dividing (ii) by (i),

$$\frac{2mg + (Mg+E)(1+k)}{2mg + Mg(1+k)} = \frac{(1+c)^2}{1}$$

or

$$\frac{[2mg + (Mg+E)(1+k)] - [2mg + Mg(1+k)]}{2mg + Mg(1+k)} = \frac{1 + c^2 + 2c - 1}{1}$$

c^2 being a small quantity is usually neglected.

$$\frac{E(1+k)}{2mg + Mg(1+k)} = 2c$$

or

$$E = \frac{2c}{(1+k)}[2mg + Mg(1+k)]$$

Effort $$\frac{E}{2} = \frac{cg}{1+k}[2m + M(1+k)] \quad (16.12)$$

- If $k = 1$,

  Effort, $$\frac{E}{2} = (m+M)cg \quad (16.13)$$

- If friction of the sleeve is considered,

  Effort, $$\frac{E}{2} = (mg + Mg + f)c$$

- For a Watt governor, $M = 0$,

  Effort, $$\frac{E}{2} = cmg \quad (16.14)$$

  Thus, the effort of a Watt governor is less than that of a Porter governor.

- Sometimes effort is defined as the force required to be applied for 1% change in speed, i.e.,

  Effort $= (m + M)cg = 0.01\,(m + M)\,g$

  In a Hartnell governor,

  $$mr\omega^2 a = \frac{1}{2}(Mg + F_s)b \quad \text{(iii)}$$

  Let E be the force applied on the sleeve to prevent its movement when the speed changes from ω to $c\omega$.

  $$mr(1+c)^2\omega^2 a = \frac{1}{2}(Mg + E + F_s)b \quad \text{(iv)}$$

Dividing (iii) by (iv),

$$\frac{1}{(1+c)^2} = \frac{Mg + F_s}{Mg + E + F_s}$$

or

$$\frac{Mg + E + F_s}{Mg + F_s} = (1+c)^2$$

or

$$\frac{E}{Mg + F_s} = 1 + c^2 + 2c - 1 = 2c \qquad \text{(neglecting } c^2\text{)}$$

or Effort,

$$\frac{E}{2} = c(Mg + F_s) \qquad (16.15)$$

POWER OF A GOVERNOR

The power of a governor is the work done at the sleeve for a given percentage change of speed, i.e., it is the product of the effort and the displacement of the sleeve.

For a Porter governor, having all equal arms which intersect on the axis or pivoted at points equidistant from the spindle axis,

$$\text{power} = \frac{E}{2} \times (2 \times \text{height of governor})$$

If the height of the governor changes from h to h_1 when the speed changes from ω to $(1 + c)\omega$,

$$h = \frac{2m + Mg(1+k)}{2m\omega^2} \quad \text{and} \quad h_1 = \frac{2m + Mg(1+k)}{2m(1+c)^2\omega^2}$$

or

$$\frac{h_1}{h} = \frac{1}{(1+c)^2}$$

$\therefore$ displacement of sleeve = $2(h - h_1)$

$$= 2h\left(1 - \frac{h_1}{h}\right)$$

$$= 2h\left(1 - \frac{1}{(1+c)^2}\right)$$

$$= 2h\left(1 - \frac{1}{1+2c}\right) \qquad \text{(neglecting } c^2\text{)}$$

$$= 2h\left(\frac{2c}{1+2c}\right)$$

$$\text{Power} = (m+M)cg \times 2h\left(\frac{2c}{1+2c}\right)$$

$$= (m+M)gh\left(\frac{4c^2}{1+2c}\right) \qquad (16.16)$$

In case $k \neq 1$,

displacement of sleeve $\approx (1 + k)(h - h_1) \approx (1 + k)\, h\left(\frac{2c}{1+2c}\right)$

and thus power $= \frac{cg}{1+k}[2m + M(1+k)] \times (1+k)h\left(\frac{2c}{1+2c}\right)$

$$= \left[m + \frac{M}{2}(1+k)\right]gh\left(\frac{4c^2}{1+2c}\right)$$

Example 16.17 ***Each ball of a Porter governor has a mass of 3 kg and the mass of the sleeve is 15 kg. The governor has equal arms, each of 200-mm length and pivoted on the axis of rotation. When the radius of rotation of the balls is 120 mm, the sleeve begins to rise up 160 mm at the maximum speed. Determine the***

(i) range of speed
(ii) lift of the sleeve
(iii) effort of the governor
(iv) power of the governor

What will be the effect of friction at the sleeve if it is equivalent to 8 N?

Solution

Refer Fig. 16.22.

$$h_1 = \sqrt{0.2^2 - 0.12^2} = 0.16 \text{ m}$$

$$h_1 = \sqrt{0.2^2 - 0.16^2} = 0.12 \text{ m}$$

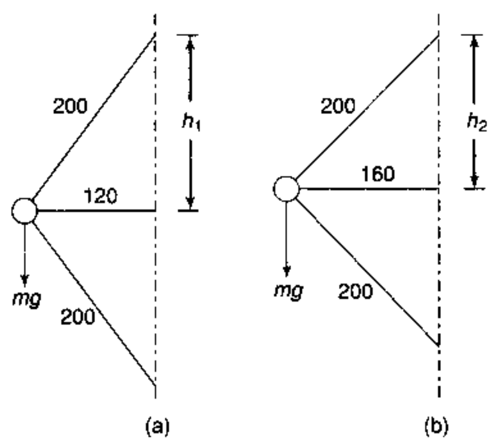


Fig. 16.22

$$N_1^2 = \frac{895}{h_1}\left(\frac{m+M}{m}\right) = \frac{895}{0.16}\left(\frac{3+15}{3}\right) = 33\,563$$

$N_1 = 183.2$ rpm

And

$$N_2^2 = \frac{895}{0.12}\left(\frac{3+15}{3}\right) = 44750 \text{ or } N_2 = 212.5 \text{ rpm}$$

or $N_2 = 212.5$ rpm

(i) Range of speed = 212.5 – 183.2 = 29.3 rpm

(ii) Lift of sleeve = $2(h_1 - h_2) = 2(0.16 - 0.12)$ = 0.08 m

(iii) Effort = $(m + M)cg$

where $c\,N = (212.5 - 183.2) = 29.2$

or $c = 29.2/183.2 = 0.16$

or Effort = (3 + 15) × 0.16 × 9.81 = 28.3 N

(iv) Power $= (m+M)gh\left(\frac{4c^2}{1+2c}\right)$

$$= (3+15) \times 9.81 \times 0.16\left(\frac{4 \times 0.16^2}{1 + 2 \times 0.16}\right)$$

= 2.26 N.m

or Power = Effort × Displacement

= 28.3 × 0.08 = 2.19 N.m

(The difference in the two values is due to the approximations taken in the derivation of relations.)

When friction is considered

$$N_1^2 = \frac{895}{h_1}\left(\frac{mg + (Mg - f)}{mg}\right)$$

$$= \frac{895}{0.16}\left(\frac{3 \times 9.81 + (15 \times 9.81 - 8)}{3 \times 9.81}\right)$$

= 32 042

N_1 = 179 rpm

$$N_2^2 = \frac{895}{h_2}\left(\frac{mg + (Mg + f)}{mg}\right)$$

$$= \frac{895}{0.12}\left(\frac{3 \times 9.81 + (15 \times 9.81 + 8)}{3 \times 9.81}\right)$$

$$= 46\,777$$

N_2 = 216.3 rpm

(iv) Range of speed = 216.3 – 179 = 37.3 rpm

(v) Lift of sleeve = Same as before = 0.08 m

Effort = $(mg + Mg + f)c$

where c = 37.3/179 = 0.208

or Effort = (3 × 9.81 + 15 × 9.81 + 8) × 0.208

= 38.4 N

(vi) Power = Effort × Displacement

= 38.4 × 0.08 = 3.07 N.m

Example 16.18 *Each ball of a Porter governor has a mass of 6 kg and the mass of the sleeve is 40 kg. The upper arms are 300 mm long and are pivoted in the axis of rotation whereas the lower arms are 250 mm long and are attached to the sleeve at a distance of 40 mm from the axis. Determine the equilibrium speed of the governor for a radius of rotation of 150 mm for 1% change in speed. Also, find the effort and the power for the same speed change.*

Solution Refer Fig. 16.23

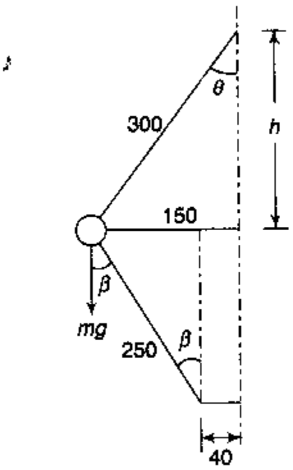


Fig. 16

$$h = \sqrt{0.3^2 - 0.15^2} = 0.26 \text{ m}$$

$$\sin\beta = \frac{150 - 40}{250} = 0.44 \text{ or } \omega = 26.1^\circ$$

$$\therefore \sin\theta = \frac{150}{300} = 0.5 \text{ or } \beta = 30^\circ$$

$$k = \frac{\tan\beta}{\tan\theta} = \frac{\tan 26.1^\circ}{\tan 30^\circ} = 0.849$$

$$h = 0.3\cos 30^\circ = 0.26 \text{ m}$$

$$N^2 = \frac{895}{h}\left(\frac{2m + M(1+k)}{2m}\right)$$

$$= \frac{895}{0.26}\left(\frac{2 \times 6 + 40(1 + 0.849)}{2 \times 6}\right)$$

$$= 24\,658$$

N = 157 rpm

$$\text{Effort} = \frac{cg}{1+k}[2m + M(1+k)]$$

$$= \frac{0.01 \times 9.81}{1 + 0.849}[2 \times 6 + 40(1 + 0.849)]$$

$$= 4.56 \text{ N}$$

$$\text{Power} = \left[m + \frac{M}{2}(1+k)\right]gh\left(\frac{4c^2}{1+2c}\right)$$

$$= \left[6 + \frac{40}{2}(1 + 0.849)\right] \times 9.81 \times 0.26\left(\frac{4 \times 0.01^2}{1 + 2 \times 0.01}\right)$$

= 42.98 × 2.55 × 0.000 392

= 0.043 N.m or 43 N.mm

Example 14.19 *In a Hartnell governor, the radius of rotation of the balls is 60 mm at the minimum speed of 240 rpm. The length of the ball arm is 130 mm and the sleeve arm is 80 mm. The mass of each ball is 3 kg and the sleeve is 4 kg. The stiffness of the spring is 20 N/mm. Determine the*

(i) speed when the sleeve is lifted by 50 mm

(ii) initial compression of the spring

(iii) governor effort

(iv) power

Solution Refer Fig. 16.24.

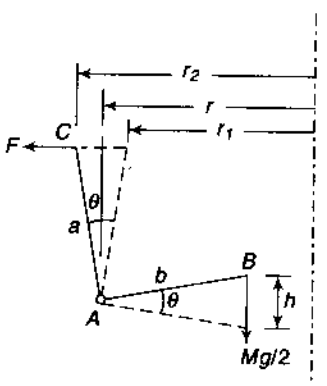


Fig. 16.24

$a = 130$ mm	$b = 80$ mm
$h = 50$ mm	$r_1 = 60$ mm
$N_1 = 240$ rpm	$s = 20\,000$ N/m
$m = 3$ kg	$M = 4$ kg

$$\omega = \frac{2\pi \times 240}{60} = 8\pi$$

$$\frac{r_2 - r_1}{a} = \theta = \frac{h}{b} \text{ or } r_2 = r_1 + \frac{ah}{b}$$

$$= 60 + \frac{130 \times 50}{80} = 141 \text{ mm}$$

(i) $s = 2 \times \dfrac{a^2}{b^2}\left(\dfrac{F_2 - F_1}{r_2 - r_1}\right)$

Now, $F_1 = mr_1\omega_1{}^2 = 3 \times 0.06 \times (8\pi)^2 = 113.7$ N

or $20\,000 = 2 \times \dfrac{0.13^2}{0.08^2}\left(\dfrac{F_2 - 113.7}{0.141 - 0.06}\right)$

$$F_2 = 113.7 + 306.7 = 420.4 \text{ N}$$

$$3 \times 0.141 \times \left(\frac{2\pi \times N_2}{60}\right)^2 = 420.4$$

$$N_2{}^2 = 90\,638$$

$$N_2 = 301 \text{ rpm}$$

(ii) $mr_1\omega_1^2 a = \dfrac{1}{2}(Mg + F_{s1})b$

$$3 \times 0.06 \times (8\pi)^2 \times 0.13 = \frac{1}{2}(4 \times 9.81 + F_{s1}) \times 0.08$$

$$F_{s1} = 330.3 \text{ N}$$

Initial compression $\dfrac{330.3}{20\,000} = 0.0165 \text{ m} = 16.5 \text{ m}$

(iii) Governor effort is also the average force applied on the spring.

$$\text{Effort} = \frac{20\,000 \times 0.05}{2} = 500 \text{ N}$$

(iv) Power = effort × displacement = 500 × 0.05 = 25 N.m

Example 16.20 ***The lengths of the ball and sleeve arms of the bell crank lever of a Hartnell governor are 140 and 120 mm respectively. The mass of each governor ball is 5 kg. The fulcrum of the bell-crank lever is at a distance of 160 mm. At the mean speed of the governor which is 270 rpm, the ball arms are vertical and the sleeve arms are horizontal. The sleeve moves up by 12 mm for an increase of speed of 4%. Neglecting friction, determine the***

(i) spring stiffness

(ii) minimum equilibrium speed when the sleeve moves by 24 mm

(iii) sensitiveness of the governor

(iv) spring stiffness for the governor to be isochronous at the mean speed

Solution Refer Fig. 16.25.

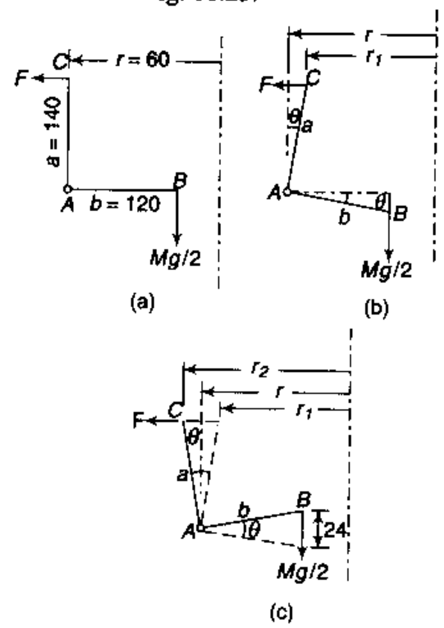


Fig. 16.25

$a = 140$ mm $\qquad b = 120$ mm

$h = 24$ mm $\qquad r = 160$ mm

$N = 270$ rpm $\qquad m = 5$ kg

$$\omega = \frac{2\pi \times 270}{60} = 9\pi \text{ rad/s}$$

$$N_2 = 1.04 \times 270 = 280.8 \text{ rpm}$$

$$\omega_2 = 1.04 \times 9\pi = 9.36\pi \text{ rad/s}$$

$$\frac{r - r_1}{a} = \theta = \frac{h}{b} \text{ or } r_1 = r - \frac{ah}{b}$$

$$= 160 - \frac{140 \times 12}{120} = 146 \text{ mm}$$

$$\frac{r - r_1}{a} = \theta = \frac{h}{b} \quad \text{or} \quad r_2 = r_1 + \frac{ah}{b}$$

$$= 146 + \frac{140 \times 24}{120} = 174 \text{ mm}$$

$$F = mr\omega^2 = 5 \times 0.16 \times (9\pi)^2 = 639.6 \text{ N}$$

$$F_2 = mr_2\omega_2^2 = 5 \times 0.174 \times (9.36\pi)^2 = 752.3 \text{ N}$$

(i) $$s = 2 \times \frac{a^2}{b^2}\left(\frac{F_2 - F}{r_2 - r}\right)$$

or $$s = 2 \times \frac{0.14^2}{0.12^2}\left(\frac{752.3 - 639.6}{0.174 - 0.16}\right)$$

$$= 21\,914 \text{ N/m or } 21.914 \text{ N/mm}$$

(ii) $$s = 2 \times \frac{a^2}{b^2}\left(\frac{F_2 - F_1}{r_2 - r_1}\right)$$

$$21\,914 = 2 \times \frac{0.14^2}{0.12^2}\left(\frac{752.3 - F_1}{0.174 - 0.146}\right)$$

$$F_1 = 752.3 - 225.4 = 526.9 \text{ N}$$

$$5 \times 0.146 \times \left(\frac{2\pi \times N_1}{60}\right)^2 = 526.9$$

$$N_1^2 = 65\,818$$

$$N_1 = 256.6 \text{ rpm}$$

(iii) Sensitiveness

$$= 2\frac{N_2 - N_1}{N_2 + N_1} = 2 \times \frac{280.8 - 256}{280.8 + 256} = 0.09 \text{ or } 9\%$$

(iv) For isochronous governor at 270 rpm,

$$F_1 = mr_1\omega^2 = 5 \times 0.146 \times (9\pi)^2 = 583.6 \text{ N}$$

$$F_2 = mr_2\omega^2 = 5 \times 0.174 \times (9\pi)^2 = 695.5 \text{ N}$$

$$s = 2 \times \frac{0.14^2}{0.12^2}\left(\frac{695.5 - 583.6}{0.174 - 0.146}\right) = 10\,880 \text{ N/m}$$

or 10.88 N/mm

CONTROLLING FORCE

When the balls of a governor rotate in their circular path, the centrifugal force on each ball tends to move it outwards. This is resisted by an equal and opposite force acting radially inwards and is known as the controlling force.

The controlling force is supplied by the weight of the rotating mass in a Watt governor, the weight of the mass and that of the sleeve in a Porter governor and by the compressed spring in the case of a Hartnell governor.

A graph showing the variation of the controlling force with the radius of rotation is called the controlling curve or diagram. This curve is useful in finding out the stability of a governor discussed below.

$$\text{Controlling force} = \tan\theta\left[mg + \frac{Mg \pm f}{2}(1 + k)\right] \qquad \text{for a Porter governor}$$

$$= \frac{1}{2}(Mg + F_s \pm f)\frac{b}{a} \qquad \text{for a Hartnell governor}$$

From the above relations, the values of the controlling force may be calculated for different radii of the ball. Figure 16.26(a) shows *AB* as the controlling force curve (neglecting friction) plotted against the ball radius. Alternatively, as the controlling force is equal and opposite to the centrifugal force, it may be computed from the relation $F = mr\omega^2$ for different radii and the corresponding speeds. This relation also indicates that for a particular speed, the controlling force is proportional to the radius. Thus a number of lines, such as *OC*,

OC_1, OC_2, etc., may be drawn on the diagram providing the values of controlling does for different radii at particular speeds. The intersection of the speed curves with the controlling force curve provides the speeds of the governor corresponding to the radii.

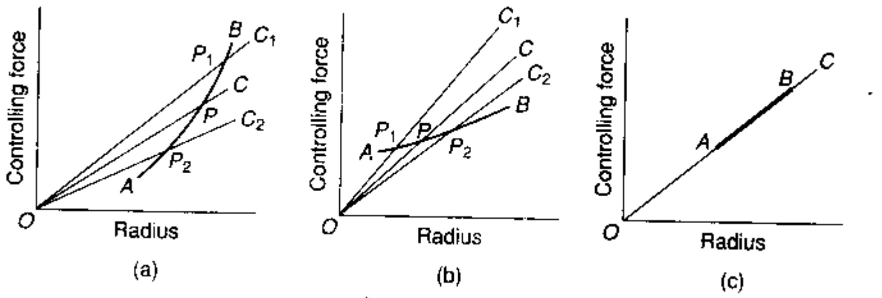


Fig. 16.26

Suppose that the point P represents the mean speed of the governor. r is the corresponding radius of the balls. Now, if the speed increases to P_1 the radius of the balls increases to r_1, thus moving the sleeve up and closing the throttle valve to the require extent. Similarly, if the speed decreases to the point P_2, the radius of the balls decreases to r_2, resulting in the lowering of the sleeve and opening the throttle valve further. This would increase the speed. This type of governor is said to be stable.

Now consider a controlling force curve of the type shown in Fig. 16.26(b). In this case, the point P again represents the mean speed of the governor. If the speed increases to P_1, the radius of the balls decreases to r_1. This means that the sleeve is lowered and the throttle valve is further opened to increase the fuel supply and consequently increasing the speed. Similarly, on decreasing the speed, the sleeve is moved up, closing the valve and thus further reducing the speed. Such a governor is therefore unstable.

Thus, for a governor to be stable the slope of the controlling force curve must be greater than that of the speed curve.

Figure 16.26(c) shows a controlling force curve AB which sometimes may be obtained in some spring-loaded governor by suitable adjustments. It can be observed that, at the speed represented by the line OC, the balls can take up any radius. Under such conditions, the governor is said to be isochronous.

If friction is taken into account, two more curves of the controlling force are obtained as shown in Fig.16.27. Thus, in all, three curves of the controlling force are obtained as follows:

(i) For steady run (neglecting friction)

(ii) While the sleeve moves up (f positive)

(iii) While the sleeve moves down (f negative)

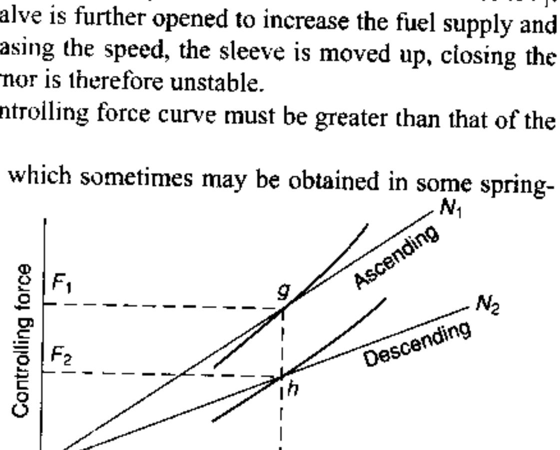


Fig. 16.27

The vertical intercept gh signifies that between the speeds corresponding to gh, the radius of the balls does not change while the direction of movement of the sleeve does. In other words, between speeds N_1 and N_2, the governor is insensitive. At all radii of the balls within the range, there are two speeds for no change of the radius.

Coefficient of Insensitiveness $\dfrac{N_1 - N_2}{N}$ is known as the coefficient of insensitiveness where N is the corresponding speed neglecting friction.

Example 16.21 ***Each arm of a Porter governor is 180 mm long and is pivoted on the axis of rotation. The mass of each ball is 4 kg and the sleeve is 18 kg. The radius of rotation of the balls is 100 mm when the sleeve begins to rise and 140 mm when at the top. Determine the range of speed. Also, find the coefficient of insensitiveness if the friction at the sleeve is 15 N.***

Solution Refer Fig. 16.28

$r_1 = 100$ mm

$h_1 = \sqrt{0.18^2 - 0.1^2} = 0.1497$ m

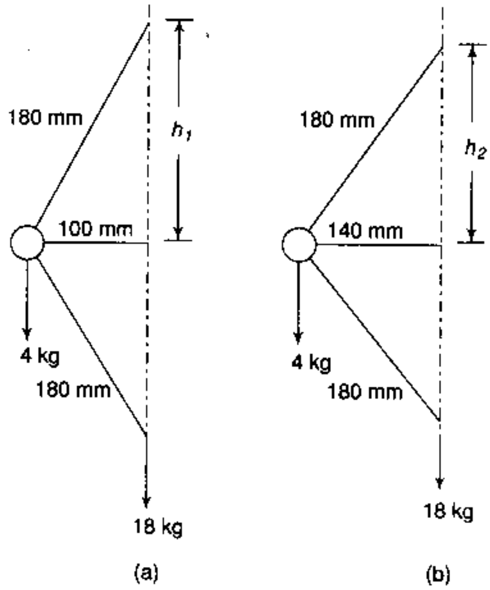


Fig. 16.28

$$N_1^2 = \frac{895}{h_1}\left(\frac{m+M}{m}\right) = \frac{895}{0.1497}\left(\frac{4+18}{4}\right)$$

$= 32\,882$

$N_1 = 181.3$ rpm

$r_2 = 140$ mm

$h_2 = \sqrt{0.18^2 - 0.14^2} = 0.1131$ m

and $N_2^2 = \dfrac{895}{0.1131}\left(\dfrac{4+18}{4}\right) = 43\,523$

or $N_2 = 208.6$ rpm

Range of speed = 208.6 – 181.3 = 27.3 rpm

Coefficient of insensitiveness

$$= \frac{N_1 - N_2}{N}$$

$$= \frac{(N_1 - N_2)}{N}\frac{(N_1 + N_2)}{(N_1 + N_2)} = \frac{(N_1^2 - N_1^2)}{2N[(N_1 + N_2)/2]}$$

$$= \frac{(N_1^2 - N_1^2)}{2N^2}$$

$$\frac{\frac{895}{h}\left(\frac{mg + (Mg + f)}{mg}\right) - \frac{895}{h}\left(\frac{mg + (Mg - f)}{mg}\right)}{\frac{895}{h}\left(\frac{m+M}{m}\right)}$$

$$= \frac{f}{(m+M)g} = \frac{15}{(4+18)\times 9.81}$$

$= 0.695$ or 6.95%

Example 16.22 ***In a Proell governor the mass of each ball is 8 kg and the mass of the sleeve is 120 kg. Each arm is 180 mm long. The length of extension of lower arms to which the balls are attached is 80 mm. The distance of pivots of arms from axis of rotation is 30 mm and the radius of rotation of the balls is 160 mm when the arms are inclined at 40° to the axis of rotation. Determine the***

(i) equilibrium speed

(ii) coefficient of insensitiveness if the friction of the mechanism is equivalent to 30 N

(iii) range of speed when the governor is inoperative

Solution Refer Fig. 16.29.

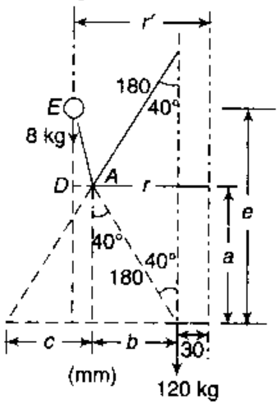


Fig. 16.29

$m = 8$ kg $\qquad r' = 160$ mm

$b = c = 180 \sin 40° = 115.7$ mm

$r = b + 30 = 115.7 + 30 = 145.7$ mm

$a = 180 \cos 40° = 137.9$ mm

$AD = r' - r = 160 - 145.7 = 14.3$ mm

$DE = \sqrt{80^2 - 14.3^2} = 78.7$ mm

$e = a + ED = 137.9 + 78.7 = 216.6$ mm

(i) Taking moments about I,

$$mr\,\omega^2 \times e = mg \times (c + r - r') + \frac{Mg}{2} \times (b + c)$$

$$8 \times 0.16 \times \omega^2 \times 0.2166 = 8 \times 9.81 \times (0.1157 + 0.1457 - 0.16)$$

$$+ \frac{120 \times 9.81}{2} \times (0.1157 + 0.1157)$$

$$0.2773\,\omega^2 = 7.958 + 136.2$$

$$\omega^2 = 519.9$$

$$\omega = 22.8 \text{ or } N = \frac{22.8 \times 30}{\pi} = 217.7 \text{ rpm}$$

(ii) Considering the friction, let ω_1 and ω_2 be the maximum and minimum speeds respectively.

$$8 \times 0.16 \times \omega_2^2 \times 0.2166 = 8 \times 9.81 \times (0.1157 + 0.1457 - 0.16)$$

$$+ \frac{120 \times 9.81 - 30}{2} \times (0.1157 + 0.1157)$$

$$0.2773\,\omega_2^2 = 7.958 + 132.7$$

$$\omega_2^2 = 507.4$$

$$\omega_2 = 22.52 \text{ or } N_2 = \frac{22.52 \times 30}{\pi} = 215.1 \text{ rpm}$$

$$0.2773\,\omega_1^2 = 7.958 + \frac{120 \times 9.81 + 30}{2} \times (0.1157 + 0.1157)$$

$$\omega_1^2 = 532.4$$

$$\omega_1^2 = 23.07 \text{ or } N_1 = \frac{23.07 \times 30}{\pi} = 220.3 \text{ rpm}$$

Coefficient of insensitiveness

$$= \frac{N_1 - N_2}{N} = \frac{220.3 - 215.1}{217.7}$$

$= 0.0239$ or 2.39%

(iii) Range of speed = 220.3 – 215.1 = 5.2 rpm

Example 16.23 ***In a spring-controlled governor the controlling force curve is a straight line. The balls are 400 mm apart when the [illegible] is 1500 N and 240 mm when [illegible] N. The mass of each ball is 10 [illegible] at which the governor [illegible] are 300 mm apart. By how [illegible] initial tension be increased to make the governor isochronous? Also, find the isochronous speed.***

Solution (i) The controlling force curve of a spring-controlling governor is a straight line and thus can be expressed as

$$F = ar + b$$

where $r = 200$ mm and $F = 1500$ N

$$1500 = 0.2\,a + b$$

When $r = 120$ mm, $\quad F = 800$ N $\qquad$ (i)

$800 = 0.12\,a + b$ $\qquad$ (ii)

From (i) and (ii),

$0.08a = 700$

or $\quad a = 8750$ and $b = -250$

$F = mr\omega^2 = ar + b$

When $r = \frac{300}{2} = 150$ mm,

$10 \times 0.15 \times \omega^2 = 8750 \times 0.15 + (-250)$

$\therefore \omega^2 = 708.3$

$$\omega = \frac{2\pi N}{60} = 26.6$$

$N = \underline{254.2 \text{ rpm}}$

(ii) To make the governor isochronous, the controlling force line must pass through the origin, i.e., b is to be zero. This is possible only if the initial tension is increased by 250 N (Refer to Fig.16.30).

(iii) $F = mr\omega^2 = ar + b$

$10 \times r \times \omega^2 = 8750\,r + 0$

$\therefore \omega^2 = 875$

$$\omega = \frac{2\pi N}{60} = 29.58$$

$N = \underline{282.5 \text{ rpm}}$

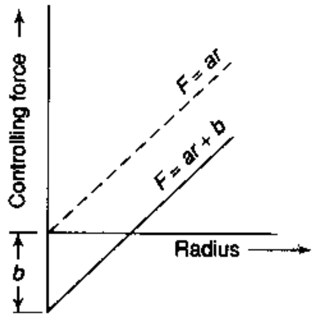


Fig. 16.30

Example 16.24 ***In a Porter governor, each arm is 200 mm long and is pivoted at the axis of rotation. The mass of each ball is 5 kg and the load on the sleeve is 30 kg. The extreme radii of rotation are 80 mm and 140 mm. Plot a graph of the controlling force vs. radius of rotation and set off a speed scale along the ordinate corresponding to a radius of 160 mm.***

Solution Controlling force of a Porter governor,

$$F = \frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1+k)\right]$$

In this case, $k = 1$ and $f = 0$

$$\therefore \quad F = \frac{r}{h}(m+M)g = \frac{(m+M)g}{\sqrt{l^2 - r^2}}r$$

We have $m = 5$ kg, $M = 30$ kg, $l = OA = 200$ mm

$$\therefore \; F = \frac{(5+30)\times 9.81}{\sqrt{(200)^2 - r^2}}r = 243.35 \times \frac{r}{\sqrt{(200)^2 - r^2}}$$

Prepare Table 16.1 for different values of r and the corresponding force.

The plot has been shown in Fig. 16.31.

To set off the speed scale,

$F = mr\omega^2$

We have $m = 5$ kg and $r = 160$ mm

$$F = 5 \times 0.160 \times \left(\frac{2\pi N}{60}\right)^2 = 0.008\ 77\, N^2$$

Now Table 16.2 can be prepared:

The speed scale can now be marked on the graph as shown in Fig. 16.31.

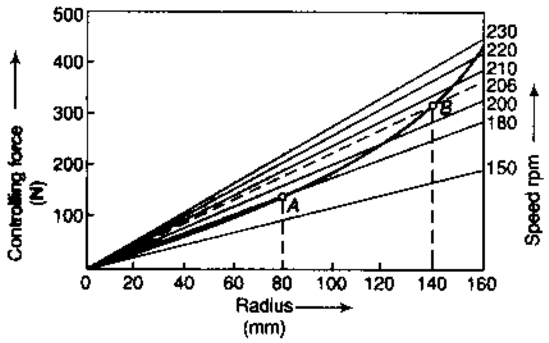


Fig. 16.31

To obtain the range of equilibrium speeds, draw vertical lines through r = 80 mm and 140 mm meeting the controlling force curve at A and B respectively. Draw straight lines from the origin and through points A and B correspond to speeds 150 and 190 rpm respectively.

The range of speed is from 180 to 206 rpm.

Table 16.1

r(mm)	20	40	60	80	100	120	140	160
F(N)	34.5	70	108	150	198	258	336	458

Table 16.2

N(rpm)	100	150	160	170	180	190	200	210	220	230
F(N)	87.7	197.3	224	253	284	317	351	387	425	464

Summary

1. The function of a governor is to maintain the speed of an engine within specified limits whenever there is variation of load.
2. The variation in the output torque of the engine during a cycle can be regulated by mounting a suitable flywheel on the shaft whereas the speed variation over a number of cycles due to load variation is regulated by governors which regulate the fuel supply according to the load.
3. The action of *centrifugal governors* depends upon the change in the centrifugal force of balls due to change in speed, and that of *inertia governors* on the acceleration or deceleration of spindle apart from change of centrifugal force.
4. The height of a *Watt governor* is inversely proportional to the square of the speed. At high speeds, the movement of the sleeve becomes very small and thus this type of governor is unsuitable for high speeds.
5. In a *Porter governor,* the sleeve is loaded with a heavy mass which improves the action of the governor.
6. Friction makes the governor inactive for a small range of speed on changing the direction of the sleeve movement.
7. In a *Proell governor,* the two balls are fixed on the upward extensions of the lower links which are in the form of bent links.
8. Further improvement in the action of governors is brought about by using springs. Spring controlled governors are *Hartnell, Hartung, Wilson Hartnell, gravity* and *Pickering.*
9. A *Wilson–Hartnell governor* uses two parallel springs alongwith an auxiliary spring.
10. A *Pickering governor* consists of three leaf springs which are arranged at equal angular intervals around the governor spindle.
11. A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of *sensitivity.*
12. *Hunting* is the process of continuous fluctuating of sleeve for longer periods whenever there is change in speed. This happens if the governor is too sensitive.
13. A governor with a zero speed range is known as an *isochronous governor.* This means that for all positions of the sleeve or the balls, the governor has the same equilibrium speed.
14. A governor is said to be *stable* if it brings the speed of the engine to the required value without much hunting and for each speed there is only one radius of rotation of the balls.
15. The *effort* of the governor is the mean force acting on the sleeve to raise or lower it for a given change of speed.
16. The *power* of a governor is the work done at the sleeve for a given percentage change of speed, i.e., it is the product of the effort and the displacement of the sleeve.
17. The centrifugal force on each ball of a governor is balanced by an equal and opposite force acting radially inwards known as *controlling force.*

Exercises

1. What is the function of a governor? How does it differ from that of a flywheel?
2. What are centrifugal governors? How do they differ from inertia governors?
3. Describe the function of a simple Watt governor. What are its limitations?
4. How does a Porter governor differ from that of a Watt governor?
5. Discuss the effect of friction on the functioning of a Porter governor? Deduce its governing equation taking into account the friction at the sleeve.
6. Describe the function of a Proell governor with the help of a neat sketch. Establish a relation among various forces acting on the bent link.
7. What are spring-controlled governors? Describe the function of any one of them.
8. Sketch a Hartnell governor. Describe its function and deduce a relation to find the stiffness of the spring.
9. Explain the working of a Hartung governor with a neat sketch.
10. Why is an auxiliary spring used along with main

springs in a Wilson–Hartnell governor? Deduce a relationship involving the stiffnesses of these springs and other parameters.

11. Describe the function of a Pickering governor or a spring-controlled gravity governor.
12. Why are the inertia governors quicker in action as compared to centrifugal governors? Explain.
13. Explain the principle of working of an inertia governor with the help of neat sketches.
14. Explain the terms *sensitiveness, hunting* and *stability* relating to governors.
15. What is the condition of isochronism in governors? In what type of governors can it be achieved? Find the required condition of isochronism in case of a Hartnell governor.
16. What is meant by effort and power of a governor? Find the expressions for the same in a Porter governor.
17. What is the controlling force of a governor? How are the controlling force curves drawn? How do they indicate the stability or instability of a governor? Indicate the shape of such a curve for an isochronous governor.
18. Figure 16.2 shows three forms of a Watt governor. In Fig. 16.2(a), length OA = 640 mm; in Fig. 16.2(b), EA = 480 mm, EF = 160 mm and in Fig.16.2(c), EA = 800 mm, EF = 160 mm. The angle $\theta = 30°$ in each case. Show that for the given configurations, the speed of rotation is the same. What will be the percentage change in speed for a 50 mm rise in the level of the balls? (4.8%; 9.8%; 1.8%)
19. Each arm of a Porter governor is 300 mm long and is pivoted on the axis of rotation. Each ball has a mass of 6 kg and the sleeve weighs 18 kg. The radius of rotation of the ball is 200 mm when the governor begins to lift and 250 mm when the speed is maximum. Determine the maximum and the minimum speeds and the range of speed of the governor. (146.8 rpm; 126.4 rpm; 20.4 rpm)
20. A Porter governor has each of its arms of 175-mm length pivoted on the axis of the governor. The radii of rotation of the balls at the minimum and the maximum speeds are 105 mm and 140 mm respectively. The mass of the sleeve is 20 kg and of each ball is 5 kg. Determine the range of speed when the friction at the sleeve is 15 N. (39.5 rpm)
21. Each arm of a Porter governor is 400 mm long. The upper arms are pivoted on the axis of the sleeve and the lower arms are attached to the sleeve at a distance of 40 mm from the axis. Each ball has a mass of 6 kg and the weight on the sleeve is 50 kg. Find the range of speed of the governor if the extreme radii of rotation of the balls are 260 mm and 300 mm. (10.6 rpm)
22. The mass of each ball of a Proell governor is 3 kg and the weight on the sleeve is 20 kg. Each arm is 220 mm long and the pivots of the upper and the lower arms are 20 mm form the axis. For the midposition of the sleeve, the extension links of the lower arms are vertical, the height of the governor is 180 mm and the speed is 150 rpm. Determine the lengths of the extension links and the tension in the upper arms. (125 mm; 155.9 N)
23. In a Hartnell governor, the lengths of the ball and the sleeve arms are equal. The extreme radii of rotation of the balls are 60 mm and 80 mm and the corresponding speeds are 160 rpm and 175 rpm. Each ball has a mass of 2 kg. Find the spring stiffness and the initial compression of the central spring. (2.01 N/mm, 33.5 mm,)
24. The following data relate to a Hartnell governor [Fig.16.12(a)]:

    m = 1.5 kg; a = 100 mm; b = 40 mm; r_1 = 70 mm; r_2 = 110 mm; N_1 = 260 rpm; and N_2 = 275 rpm.

    The axis of rotation is 80 mm from the fulcrum. Calculate the rate of the spring and the equilibrium speed when the radius of the balls is 80 mm. (18.44 N/mm; 265.3 rpm)
25. The following data refer to a Wilson–Hartnell governor:

    Mass of each ball = 2.5 kg
    Minimum speed = 210 rpm
    Maximum speed = 220 rpm
    Minimum radius = 120 mm
    Maximum radius = 160 mm
    Length of ball arm of each bell-crank lever = 180 mm
    Length of sleeve arm of each bell-crank lever = 120 mm
    Combined stiffness of the two ball-springs = 300 N/m

    Determine the equivalent stiffness of the auxiliary spring referred to the sleeve.
    [Equivalent stiffness of the auxiliary spring referred to the sleeve = $S_a\,(y/x)^2$] (11.22 kN/m)
26. A gramophone is driven by a Pickering governor, each spring of which is 4 mm wide and 0.1 mm thick with a length of 40 mm. The distance between the spindle axis and the centre of mass when the governor is at rest is 6 mm. A mass of 20 g is attached to each leaf spring at the centre. Determine the speed of the turn table for a sleeve

lift of 0.5 mm if the ratio of the governor speed to the turn table speed is 8. E = 205 GN/m^2.

(68.9 rpm)

27. The mass of each ball of a spring-controlled gravity governor is 1.4 kg. The bell-crank lever has a 90-mm long vertical arm and a 40-mm long horizontal arm. The distance of the fulcrum from the axis of rotation is 45 mm. The sleeve has a mass of 7.5 kg. The sleeve begins to rise at 220 rpm and the rise of sleeve for 6% rise is 8 mm. Find the initial thrust in the spring and its stiffness.

(49.45 N; 9.19 N/mm)

28. A Porter governor has equal arms. Each arm is 240 mm long and is pivoted on the axis of rotation. Each ball has a mass of 5 kg and the load on the sleeve is 18 kg. The ball radius is 150 mm when the sleeve begins to rise and 200 mm at the maximum speed. Find the range of speed. Also, determine the coefficient of insensitiveness if the friction at the sleeve is equivalent to a force of 10 N.

(27.9 rpm; 0.044)

29. Each arm of a Porter governor is 250 mm long and is pivoted on the axis of rotation. The mass of each ball is 5 kg and the sleeve is 25 kg. The sleeve begins to rise when the radius of rotation of the balls is 150 mm and reaches the top when it is 200 mm. Determine the range of speed, lift of the sleeve, governor effort and power. In what way are these values changed if friction at the sleeve is equivalent to 10 N?

(25 rpm, 0.1 m, 44.8 N, 4.48 N.m;
31.3 rpm, 0.1 m, 57.3 N, 5.73 N.m)

30. The controlling force in a spring controlled governor is 1500 N when the radius of rotation of the balls is 200 mm and 887.5 N when it is 130 mm. The mass of each ball is 8 kg. If the controlling force curve is a straight line, determine the controlling force and the speed of rotation when the radius of rotation is 150 mm. Also find the increase in the initial tension so that the governor is isochronous. What will be the isochronous speed?

(1063 N, 284 rpm, 250 N, 316 rpm)

17 GYROSCOPE

Introduction

If the axis of a spinning or rotating body is given an angular motion about an axis perpendicular to the axis of spin, an angular acceleration acts on the body about the third perpendicular axis. The torque required to produce this acceleration is known as the *active gyroscopic torque*. A reactive gyroscopic torque or couple also acts similar to the concept of centripetal and centrifugal forces on a rotating body. The effect produced by the reactive gyroscopic couple is known as the *gyroscopic effect*. Thus aeroplanes, ships, automobiles, etc., that have rotating parts in the form of wheels or rotors of engines experience this effect while taking a turn, i.e., when the axes of spin is subjected to some angular motion.

17.1 ANGULAR VELOCITY

The angular velocity of a rotating body is specified by

- the magnitude of velocity
- the direction of the axis of rotor
- the sense of rotation of the rotor, i.e., clockwise or counter-clockwise

Angular velocity is represented by a vector in the following manner:

(i) Magnitude of the velocity is represented by the length of the vector.

(ii) Direction of axis of the rotor is represented by drawing the vector parallel to the axis of the rotor or normal to the plane of the angular velocity.

(iii) Sense of rotation of the rotor is denoted by taking the direction of the vector in a set rule. The general rule is that of a right-handed screw, i.e., if a screw is rotated in the clockwise direction, it goes away from the viewer and vice-versa.

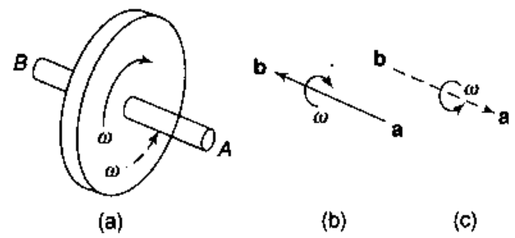


Fig. 17.1

For example, Fig.17.1(a) shows a rotor which rotates in the clockwise direction when viewed from the end A. Its angular motion has been shown vectorially in Fig.17.1(b). The vector has been taken to a scale parallel to the axis of the rotor. The sense of direction of the vector is from **a** to **b** according to the screw rule. However, if the direction of rotation of the rotor is reversed, it would be from **b** to **a** [Fig.17.1(c)].

17.2 ANGULAR ACCELERATION

Let a rotor spin (rotate) about the horizontal axis Ox at a speed of ω rad/s in the direction as shown in Fig.17.2(a). Let **oa** represent its angular velocity [Fig.17.2(b)].

Now, if the magnitude of the angular velocity changes to $(\omega + \delta\omega)$ and the direction of the axis of spin to Ox' (in time δt), the vector **ob** would represent its angular velocity in the new position. Join **ab** which represents the change in the angular velocity of the rotor. The vector **ab** can be resolved into two components:

(i) **ac** representing angular velocity change in a plane normal to **ac** or x-axis, and

(ii) **cb** representing angular velocity change in a plane normal to **cb** or y-axis.

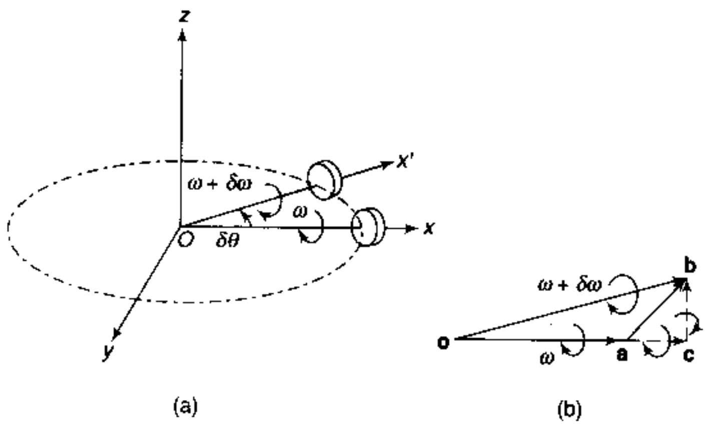


Fig. 17.2

Change of angular velocity, **ac** $= (\omega + \delta\omega)\cos\delta\theta - \omega$

Rate of change of angular velocity $= \dfrac{(\omega + \delta\omega)\cos\delta\theta - \omega}{\delta t}$

$\therefore$ angular acceleration $= \underset{\delta t \to 0}{\text{Lt}} \dfrac{(\omega + \delta\omega)\cos\delta\theta - \omega}{\delta t}$

As $\delta t \to 0$, $\delta\theta \to 0$ and $\cos\delta\theta \to 1$

$\therefore$ angular acceleration $= \underset{\delta t \to 0}{\text{Lt}} \dfrac{\omega + \delta\omega - \omega}{\delta t} = \dfrac{d\omega}{dt}$

Change of angular velocity, **cb** $= (\omega + \delta\omega)\sin\delta\theta$

Rate of change of angular velocity $= \dfrac{(\omega + \delta\omega)\sin\delta\theta}{\delta t}$

$\therefore$ angular acceleration $= \underset{\delta t \to 0}{\text{Lt}} \dfrac{(\omega + \delta\omega)\sin\delta\theta}{\delta t}$

As $\delta t \to 0$, $\delta\theta \to 0$ and $\sin\delta\theta \to \delta\theta$

$\therefore$ angular acceleration $= \underset{\delta t \to 0}{\text{Lt}} \dfrac{(\omega + \delta\omega)\delta\theta}{\delta t} = \omega\dfrac{d\theta}{dt}$

Total angular acceleration, $\alpha = \dfrac{d\omega}{dt} + \omega\dfrac{d\theta}{dt}$ (17.1)

This shows that the total angular acceleration of the rotor is the sum of

(i) $d\omega/dt$, representing change in the magnitude of the angular velocity of the rotor

(ii) $\omega.d\theta/dt$, representing change in the direction of the axis of spin, the direction of **cb** is from **c** to **b** in the vector diagram (being a component of **ab**), the acceleration acts clockwise in the vertical plane xz (when viewed from front along the y-axis)

GYROSCOPIC TORQUE (COUPLE)

Let I be the moment of inertia of a rotor and ω its angular velocity about a horizontal axis of spin Ox in the direction as shown in Fig.17.3(a). Let this axis of spin turn through a small angle $\delta\theta$ in the horizontal plane (xy) to the position Ox' in time δt.

Figure 17.3(b) shows the vector diagram. **oa** represents the angular velocity vector when the axis is Ox and **ob** when the axis is changed to Ox'. Then **ab** represents the change in the angular velocity due to change in direction of the axis of spin of the rotor. This change in the angular velocity is clockwise when viewed from **a** towards **b** and is in the vertical plane xz. This change results in an angular acceleration, the sense and direction of which are the same as that of the change in the angular velocity.

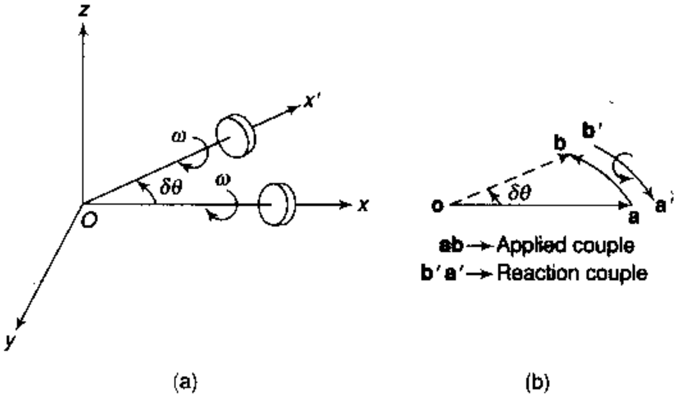


Fig. 17.3

Change in angular velocity, **ab** = $\omega \times \delta\theta$

Angular acceleration, $\alpha = \omega \dfrac{\delta\theta}{\delta t}$

In the limit, when $\delta t \to 0$, $\alpha = \omega \dfrac{\delta\theta}{\delta t}$

Usually, $d\theta/dt$, the angular velocity of the axis of spin is called the *angular velocity of precession* and is denoted by ω_p.

$\therefore$ Angular acceleration, $\alpha = \omega.\omega_p$.

The torque required to produce this acceleration is known as the gyroscopic torque and is a couple which must be applied to the axis of spin to cause it to rotate with angular velocity ω_p about the axis of precession Oz.

$$\text{Acceleration torque, } T = I\alpha = I\,\omega\,\omega_p \tag{17.2}$$

For the configuration of Fig.17.3(a),

Ox is known as the axis of spin

Oz is known as the axis of precession

Oy is known as the axis of gyroscopic couple

yz is the plane of spin (parallel to plane of rotor)

xy is the plane of precession

yz is the plane of gyroscopic couple00

A gyroscope

The torque obtained above is that which is required to cause the axis of spin to precess in the horizontal plane and is known as the *active gyroscopic torque* or the applied torque. A *reactive gyroscopic torque* or reaction torque is also applied to the axis which tends to rotate the axis of spin in the opposite direction, i.e., in the counter-clockwise direction in the above case. Just as the centrifugal force on a rotating body tends to move the body outwards, while a centripetal acceleration (and thus centripetal force) acts on it inwards, in the same way, the effects of active and reactive gyroscopic torques can be understood.

The effect of the gyroscopic couple on a rotating body is known as the *gyroscope effect* on the body. A *gyroscope* is a spinning body which is free to move in other directions under the action of external forces.

Example 17.1 *A uniform disc having a mass of 8 kg and a radius of gyration of 150 mm is mounted on one end of a horizontal arm of 200-mm length. The other end of the arm can rotate freely in a universal bearing. The disc is given a clockwise spin of 250 rpm as seen from the disc end of the arm. Determine the motion of the disc if the arm remains horizontal.*

Solution

$m = 8$ kg $\qquad l = 0.2$ m

$k = 0.15$ m $\qquad N = 240$ rpm

$I = mk^2 = 8 \times (0.15)^2 = 0.18$ kg.m^2

$$\omega = \frac{2\pi \times 240}{60} = 25.13 \text{ rad/s}$$

$C = I\,\omega\omega_p$

$Mgl = I\,\omega\,\omega_p$

$8 \times 9.81 \times 0.2 = 0.18 \times 25.13 \times \omega_p$

$\omega_p = 3.47$ rad/s

As the disc rotates, the weight of the disc acts downwards and thus a couple about the y-axis, in the clockwise direction is applied on the disc (Fig.17.4). The reaction couple is thus counter-clockwise which tends to keep the arm horizontal.

Assuming that the axis of spin Ox precesses about the z-axis in the counter-clockwise direction, the vector **oa** would rotate to the position **ob** in a short period. Then **ab** is the applied couple and **b′a′** is the reaction couple which is clockwise and tends to give the arm a clockwise rotation about the y-axis which is not true. Thus, the axis of spin Ox must precess about the z-axis in the clockwise direction with an angular velocity of 3.47 rad/s.

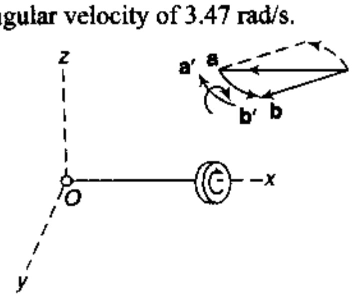


Fig. 17.4

Example 17.2 *A disc with radius of gyration of 60 mm and a mass of 4 kg is mounted centrally on a horizontal axle of 80 mm length between the bearings. It spins about the axle at 800 rpm counter-clockwise when viewed from the right-hand side bearing. The axle precesses about a ver axis at 50 rpm in the clockwise direction when viewed from above. Determine the resultant reaction at each bearing due to the mass and the gyroscopic effect.*

Solution

$m = 4$ kg $\qquad N = 800$ rpm

$k = 0.06$ m $\qquad N_p = 50$ rpm

$I = mk^2 = 4 \times (0.06)^2$

$= 0.0144$ kg.m^2

$l = 80$ mm $= 0.08$ m

$$\omega = \frac{2\pi \times 800}{60} = 83.78 \text{ rad/s}$$

$$\omega_p = \frac{2\pi \times 50}{60} = 5.24 \text{ rad/s}$$

$\therefore \quad C = I\,\omega\omega_p$

$= 0.0144 \times 83.78 \times 5.24 = 6.32$ N.m

The applied (active) and reaction couples are shown in Fig. 17.5. The reaction couple is clockwise when viewed from front and tends to raise the bearing A and lower the bearing B. Thus, reaction of each bearing in turn is downwards at A and upwards at B.

Reaction at bearing A due to gyro. couple

$$= \frac{C}{l} = \frac{6.32}{0.08} = 79 \text{ N (downwards)}$$

Reaction at bearing B due to gyro. couple = 79 N (upwards)

Force at each bearing due to weight of the disc

$$= \frac{4 \times 9.81}{2} = 19.6 \text{ N}$$

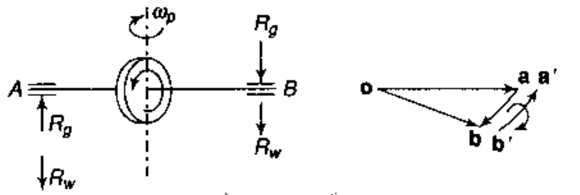


Fig. 17.5

or Reaction at each bearing due to weight = 19.6 N (upwards)

∴ Reaction at bearing A = 79 − 19.6 = 59.4 N (upwards)

Reaction at bearing B = 79 + 19.6 = 98.6 N (downwards)

17.4 GYROSCOPIC EFFECT ON AEROPLANES

Figure 17.6(a) shows an aeroplane in space. Let the propeller be rotating in the clockwise direction when viewed from the rear end. The angular momentum vector **oa** due to the angular velocity is shown in Fig. 17.6(b).

- If the plane takes a left turn, the angular momentum vector is shifted and may be represented by the vector **ob**. The change is shown by the vector **ab** and is the active gyroscopic couple. This vector is in the horizontal plane and is perpendicular to the vector **oa** in the limit. The reactive vector is given by **b'a'** which is equal and opposite to the vector **ab**. The interpretation of this vector shows that the couple acts in the vertical plane and is counter-clockwise when viewed from the right-hand side of the plane. This indicates that it tends to raise the nose and depress the tail of the aeroplane.
- Figure 17.6(c) shows the gyroscopic effect, when the aeroplane takes the right turn. The change is shown by the vector **cd** and is the active gyroscopic couple. It is perpendicular to the vector **oc** in the limit in the horizontal plane. The reactive couple is given by **d'c'**. The couple acts in the vertical plane and is clockwise when viewed from the right-hand side of the plane. Thus, it tends to dip the nose and raise the tail of the aeroplane.
- If the rotation of the engine is reversed, i.e., it rotates counter-clockwise when viewing from the rear end, the angular momentum vector is **oe** as shown in Fig. 17.6(d). On taking a left turn, it changes to **of**. The active gyroscopic vector is **ef** and the reactive **fe'**. Viewing from the right-hand side of the plane, it indicates that the nose is dipped and the tail is raised. Similarly, when the plane takes a right turn, the effect is indicated in Fig. 15.6(e). The nose is raised and the tail is depressed.

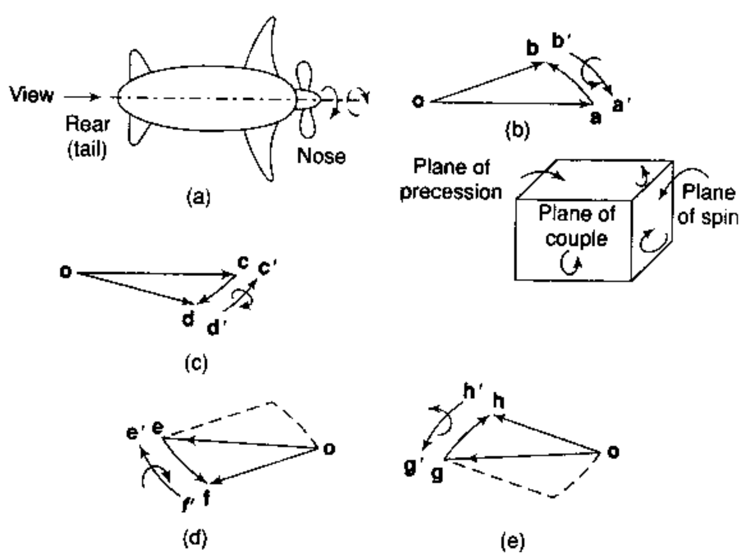


Fig. 17.6

It can be concluded from the above cases that if the direction of either the spin of the rotor or of the precession is changed, the gyroscopic effect is reversed, but if both are changed, the effect remains the same.

Example 17.3 *An aeroplane flying at 240 km/h turns towards the left and completes a quarter circle of 60m radius. The mass of the rotary engine and the propeller of the plane is 450 kg with a radius of gyration of 320 mm. The engine speed is 2000 rpm clockwise when viewed from the rear. Determine the gyroscopic couple on the aircraft and state its effect.*

In what way is the effect changed when the

(i) aeroplane turns towards right

(ii) engine rotates clockwise when viewed from the front (nose end) and the aeroplane turns (a) left (b) right?

Solution

$m = 450$ kg, $k = 0.32$ m,

$$\omega = \frac{2\pi \times 2000}{60} = 209.4 \text{ rad/s},$$

$$v = \frac{240 \times 10^3}{3600} = 66.67 \text{ m/s}$$

$$I = mk^2 = 450 \times (0.32)^2 = 46.08 \text{ kg.m}^2$$

$$\omega_p = \frac{v}{r} = \frac{66.67}{60} = 1.11 \text{ rad/s}$$

$$C = I\,\omega\omega_p = 46.08 \times 209.4 \times 1.11$$

$$= 10\,713 \text{ N.m} = 10.713 \text{ kN.m}$$

The effects of gyroscopic couples on the aeroplane when it takes a left or right turn are discussed in the previous section. In brief it is summarized below:

Figure 17.6(a) shows the aeroplane in space. When it turns left,

oa is the angular momentum vector before turning [Fig.17.6(b)];

ob is the angular momentum vector after turning;

ab is the applied or active gyroscopic couple;

b′a′ is the reaction couple and is perpendicular to **oa** in the limit. This couple acts in the vertical plane and tends to raise the nose and depress the tail of the aeroplane.

(i) If the aeroplane takes a right turn, the reaction couple is **d′c′** [Fig. 17.6(c)], which shows that the nose is depressed and the tail is raised.

(ii) (a) When the engine rotates or spins clockwise on viewing from the nose end, **ef** is the applied couple as the aeroplane turns left [Fig.17.6(d)]. **f′e′** is the reaction couple indicating that the nose is depressed and the tail is raised.

(b) When the aeroplane takes a right turn, the tail is depressed and the nose is raised [Fig.17.6(e)].

17.5 GYROSCOPIC EFFECT ON NAVAL SHIPS

Some of the terms used in connection with the motion of naval ships or sea vessels are given below [Fig.17.7(a)]:

(a) *Bow* is the fore or the front end.

(b) *Stern* or *aft* is the rear end.

(c) *Starboard* is the right-hand side when looking from the stern.

(d) *Port* is the left-hand side when looking from the stern.

(e) *Steering* is turning on the side when viewing from the top.

(f) *Pitching* is limited angular motion of the ship about the transverse axis.

(g) *Rolling* is limited angular motion of the the ship about the longitudinal axis.

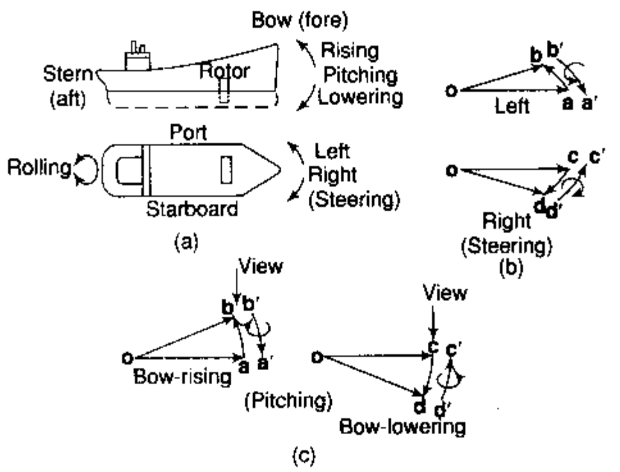


Fig. 17.7

Let the plane of spin of the rotor and other rotating masses be horizontal and across the breadth of the ship. Assume ω to be the angular velocity of the rotor in the clockwise direction when viewed from stern (rear end).

Gyroscopic Effect during Pitching

When the ship turns left, the angular momentum vector changes from **oa** to **ob** [Fig.17.7 (b)]. The reaction couple is found to be **b′a′** which tends to raise the bow and lower the stern. On turning right, the reaction couple is reversed so that bow is lowered and the stern is raised.

Gyroscopic Effect on Pitching

Pitching of the ship is usually considered to take place with simple harmonic motion. A simple harmonic motion is represented by, $x = X \sin \omega_0 t$.

Such a motion is obtained by the projection of a rotating vector X on a diameter while rotating around a circle with a constant angular velocity ω_0 and where x is the displacement from the mean position in time t. In the same way, angular displacement θ of the axis of spin from its mean position is given by

$$\theta = \varphi \sin \omega_0 t$$

where φ = amplitude (angular) of swing or the maximum angle turned from the mean position in radius

ω_0 = angular velocity of SHM $\dfrac{2\pi}{\text{Time period}}$

Angular velocity of precession, $\dfrac{d\theta}{dt} = \varphi\omega_0 \cos \omega_0 t$

This is maximum when $\cos \omega_0 t = 1$.

Therefore, maximum angular velocity of precession $\omega_p = \varphi\, \omega_0$

$$\text{Gyroscopic couple } I\omega\omega_p = I\omega\left(\varphi \times \frac{2\pi}{\text{Time period}}\right) \tag{17.4}$$

When the bow is rising, the reaction couple is clockwise on viewing from top and thus the ship would move towards right or starboard side. Similarly, when the bow is lowered, the ship turns towards left or port side [Fig. 17.7(c)].

$$\text{Angular acceleration} = \varphi\omega_0^2 \sin \omega_0 t \tag{17.5}$$

$$\text{Maximum angular acceleration} = \varphi\omega_0^2 \tag{17.6}$$

Gyroscopic Effect on Rolling

As the axes of the rolling of the ship and of the rotor are parallel, there is no precession of the axis of spin and thus there is no gyroscopic effect.

In the same way, the effects on steering, pitching or rolling can be observed when the plane of the spin of the rotating masses is horizontal but along the longitudinal axis of the vessel or when the axis is vertical.

Example 17.4 *The turbine rotor of a ship has a mass of 2.2 tonnes and rotates at 1800 rpm clockwise when viewed from the aft. The radius of gyration of the rotor is 320 mm. Determine the gyroscopic couple and its effect when the*

(i) ship turns right at a radius of 250 m with a speed of 25 km/h

(ii) ship pitches with the bow rising at an angular velocity of 0.8 rad/s

(iii) ship rolls at an angular velocity of 0.1 rad/s

Solution

$m = 2200$ kg $N = 1800$ rpm

$R = 250$ m $v = 25$ km/h

$k = 0.32$ m $= \frac{25 \times 1000}{3600}$

$= 6.94$ m/s

$I = mk^2 = 2200 \times (0.32)^2$

$= 225.3$ kg.m^2

$$\omega = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/s}$$

$$\omega_p = \frac{v}{R} = \frac{6.94}{250} = 0.0278 \text{ rad/s}$$

(i) $C = I\omega\omega_p$

$= 225.3 \times 188.5 \times 0.0278$

$= 1180$ N.m

The effect is to lower the bow (fore) and raise the stern (aft) when the ship turns right [Fig. 17.7(b)].

(ii) $\omega_p = 0.8$ rad/s

$C = I\omega\omega_p = 225.3 \times 188.5 \times 0.8$

$= 33\,972$ N.m

The effect of the reaction couple when the bow is rising, is to turn the ship towards right or towards starboard.

(iii) $\omega_p = 0.1$ rad/s

$C = 225.3 \times 188.5 \times 0.1 = 4246.5$ N.m

As the axis of spin is always parallel to the axis of precession for all positions, there is no gyroscopic effect on the ship.

Example 17.5 *The rotor of the turbine of a ship has a mass of 2500 kg and rotates at a speed of 3200 rpm counter-clockwise when viewed from stern. The rotor has radius of gyration of 0.4 m. Determine the gyroscopic couple and its effect when*

(i) the ship steers to the left in a curve of 80-m radius at a speed of 15 knots (1 knot = 1860 m/h)

(ii) the ship pitches 5 degrees above and 5 degrees below the normal position and the bow is descending with its maximum velocity—the pitching motion is simple harmonic with a periodic time of 40 seconds

(iii) the ship rolls and at the instant, its angular velocity is 0.4 rad/s clockwise when viewed from stern

Also find the maximum angular acceleration during pitching.

Solution

$m = 2500$ kg $N = 1800$ rpm

$k = 0.4$ m $v = \frac{15 \times 1860}{3600} = 7.75$ m/s

$I = mk^2 = 2500 \times (0.4)^2 = 400$ kg.m^2

$$\omega = \frac{2\pi \times 3200}{60} = 335 \text{ rad/s}$$

(i) $R = 80$ m

$$\omega_p = \frac{v}{R} = \frac{775}{80} = 0.097 \text{ rad/s}$$

$C = 400 \times 335 \times 0.097$

$= 12\,981$ N.m

The effect is to lower the bow and raise the stern [Figs 17.8 (a) and (b)].

(ii) $\varphi = 5° = 5 \times \frac{\pi}{180} = 0.0873$ rad

$T = 40$ s

$$\therefore \ \omega_0 = \frac{2\pi}{40} = 0.157 \text{ rad/s}$$

$\omega_p = \varphi\,\omega_0 = 0.0873 \times 0.157 = 0.0137$ rad/s

$C = I\,\omega\,\omega_p = 400 \times 335 \times 0.0137 = 1837.5$ N.m

As the bow descends during pitching, the ship would turn towards right or starboard [Figs 17.8(a) and (c)].

(iii) $\omega_p = 0.04$ rad/s

$C = 400 \times 335 \times 0.04 = 5360$ N.m

No gyroscopic effect is there as discussed earlier.

Maximum angular acceleration during pitching

$\alpha_{max} = \varphi\omega_0^2 = 0.0873 \times (0.157)^2 = \underline{0.00215 \text{ rad/s}^2}$

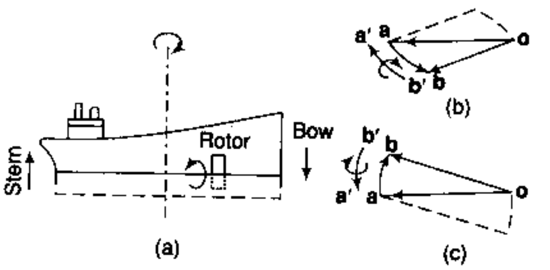


Fig. 17.8

17.6 STABILITY OF AN AUTOMOBILE

In case of a four-wheeled vehicle, it is essential that no wheel is lifted off the ground while the vehicle takes a turn. The condition is fulfilled as long as the vertical reaction of the ground on any of the wheels is positive (or upwards).

Figure 17.9 shows a four-wheeled vehicle having a mass m. Assuming that the weight is equally divided among the four wheels,

$$\text{weight on each wheel} = \frac{W}{4} = \frac{mg}{4} \text{ (downwards)}$$

$$\text{Reaction of ground on each wheel, } R_w = \frac{W}{4} = \frac{mg}{4} \text{ (upwards)}$$

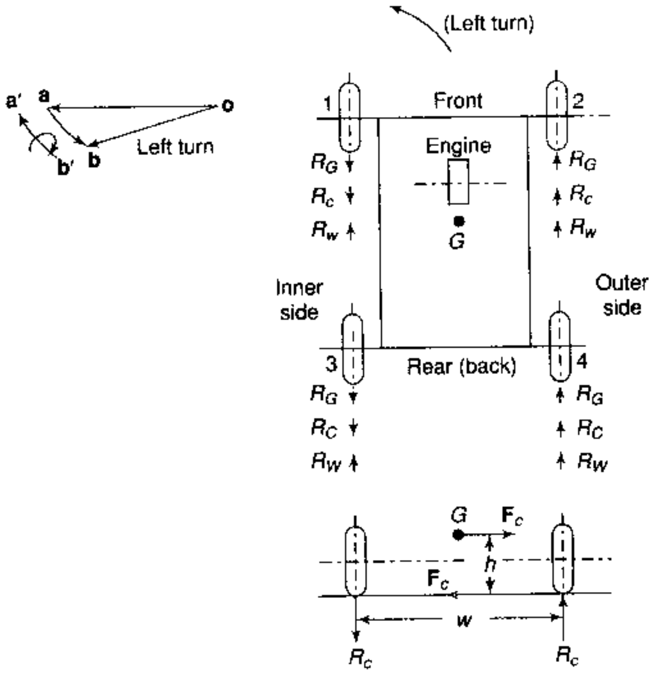


Fig. 17.9

Effect of Gyroscopic Couple

$$\text{Gyroscopic couple due to four wheels, } C_w = 4I_w\omega_w\omega_p = 4I_w\frac{v^2}{rR}$$

where I_w = mass moment of inertia of each wheel

ω_w = angular velocity of wheels = $\frac{v}{r}$

ω_p = angular velocity of precession = $\frac{v}{R}$

v = linear velocity of the vehicle

R = radius of curvature

Gyroscopic couple due to engine rotating parts,

$$C_e = I_e\, \omega_e\, \omega_p = I_e\, G\, \omega_w\, \omega_p$$

where G is the gear ratio $= \dfrac{\omega_e}{\omega_w}$

Total gyroscopic couple, $C_G = C_w \pm C_e$

Positive sign is used when the engine parts rotate in the same direction as the wheels and the negative sign when they rotate in the opposite.

Assuming that C_G is positive and the vehicle takes a left turn, the reaction gyroscopic couple on it is clockwise when viewed from the rear of the vehicle. The reaction couple is provided by equal and opposite forces on the outer and the inner wheels of the vehicle.

Forces on the two outer wheels $= \dfrac{C_G}{w}$ (downwards)

Forces on the two inner wheels $= \dfrac{C_G}{w}$ (upwards)

Forces on each of the outer wheels $= \dfrac{C_G}{2w}$ (downwards)

Forces on each of the inner wheels $= \dfrac{C_G}{2w}$ (upwards)

Thus the force on each of the outer wheels is similar to the weight. On the inner wheels it is in the opposite direction. Thus,

Reaction of ground on each outer wheel, $R_G = \dfrac{C_G}{2w}$ (upwards)

Reaction of ground on each inner wheel, $R_G = \dfrac{C_G}{2w}$ (downwards)

Effect of Centrifugal Couple

As the vehicle moves on a curved path, a centrifugal force also acts on the vehicle in the outward direction at the centre of mass of the vehicle.

$$\text{Centrifugal force, } mR\omega_p^2 = mR\left(\frac{v}{R}\right)^2 = m\frac{v^2}{R}$$

This force would tend to overturn the vehicle outwards and the overturning couple will be

$$C_c = mR\omega_p^2 \times h = m\frac{v^2}{R}h$$

This is equivalent to a couple due to equal and opposite forces on outer and inner wheels.

Force on each outer wheel $= \dfrac{C_c}{2w}$ (downwards)

Force on each inner wheel $= \dfrac{C_c}{2w}$ (upwards)

Again, the force on each of the outer wheels is similar to the weight and on each of the inner wheels, it is opposite.

Reaction of ground on each outer wheel, $R_c = \dfrac{C_c}{2w}$ (upwards)

Reaction of ground on each inner wheel, $R_c = \frac{C_c}{2w}$ (downwards)

Vertical reaction on each outer wheel $= \frac{W}{4} + \frac{C_G}{2w} + \frac{C_c}{2w}$ (upwards)

Vertical reaction on each inner wheel $= \frac{W}{4} + \frac{C_G}{2w} + \frac{C_c}{2w}$ (upwards)

It can be observed that there are chances that the reaction of the ground on the inner wheels may not be upwards and thus the wheels are lifted from the ground. For positive reaction, the conditions will be

$$\frac{W}{4} - \frac{C_G}{2w} - \frac{C_c}{2w} \geq 0$$

or
$$\frac{W}{4} \geq \frac{C_G + C_c}{2w}$$

or
$$R_w \geq R_G + R_c \tag{17.7}$$

Example 17.6 ***Each wheel of a four-wheeled rear engine automobile has a moment of inertia of 2.4 kg.m² and an effective diameter of 660 mm. The rotating parts of the engine have a moment of inertia of 1.2 kg.m². The gear ratio of engine to the back wheel is 3 to 1. The engine axis is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The mass of the vehicle is 2200 kg and the centre of the mass is 550 mm above the road level. The track width of the vehicle is 1.5 m. Determine the limiting speed of the vehicle around a curve with 80 m radius so that all the four wheels maintain contact with the road surface.***

Solution

$I_w = 2.4$ kg.m^2	$m = 2200$ kg
$r = 0.33$ m	$h = 0.55$ m
$I_e = 1.2$ kg.m^2	$w = 1.5$ m
$G = \frac{\omega_e}{\omega_w} = 3$	$R = 80$ m

(i) Reaction due to weight

$$R_w = \frac{mg}{4} = \frac{2200 \times 9.81}{4} = 5395.5 \text{ N (upwards)}$$

(ii) Reaction due to gyroscopic couple

$$C_w = 4I_w \frac{v^2}{rR} = 4 \times 2.4 \times \frac{v^2}{0.33 \times 80} = 0.364v^2$$

$$C_e = I_e G \omega_w \omega_p = 1.2 \times 3 \times \frac{v^2}{0.33 \times 80} = 0.136v^2$$

$\therefore$ $C_G = C_w + C_e = 0.364\ v^2 + 0.136\ v^2 = 0.5\ v^2$

Reaction on each outer wheel,

$$R_{Go} = \frac{C_G}{2w} = \frac{0.5v^2}{2 \times 1.5} = 0.167v^2 \text{ (upwards)}$$

Reaction on each inner wheel,

$R_{Gi} = 0.167\ v^2$ (downwards)

(iii) Reaction due to centrifugal couple

$$C_c = \frac{mv^2}{R}h = \frac{2200 \times v^2}{80} \times 0.55 = 15.125v^2$$

Reaction on each outer wheel,

$$R_{co} = \frac{C_c}{2w} = \frac{15.125v^2}{2 \times 1.5} = 5.042v^2 \text{ (upwards)}$$

Reaction on each inner wheel,

$R_{ci} = 5.042\ v^2$ (downwards)

For maximum safe speed, the condition is

$R_w = R_{Gi} + R_{ci}$

$5395.5 = (0.167 + 5.042)\ v^2$

$v^2 = 1035.8$

$v = 32.18$ m/s

or $v = \frac{32.18 \times 3600}{1000} = 115.9$ km/h

Example 17.7 *A four-wheeled trolley car has a total mass of 3000 kg. Each axle with its two wheels and gears has a total moment of inertia of 32 kg.m². Each wheel is of 450-mm radius. The centre distance between two wheels on an axle is 1.4 m. Each axle is driven by a motor with a speed ratio of 1:3. Each motor along with its gear has a moment of inertia of 16 kg.m² and rotates in the opposite direction to that of the axle. The centre of mass of the car is 1 m above the rails. Calculate the limiting speed of the car when it has to travel around a curve of 250-m radius without the wheels leaving the rails.*

Solution

$I_w = \frac{32}{2} = 16 \text{ kg.m}^2$ $\quad m = 3000$ kg

$r = 0.45$ m $\quad h = 1$ m

$I_m = 16 \text{ kg.m}^2$ $\quad w = 1.4$ m

$R = 250$ m

(i) Reaction due to weight

$$R_w = \frac{mg}{4} = \frac{3000 \times 9.81}{4} = 7357.5 \text{ N (upwards)}$$

(ii) Reaction due to gyroscopic couple

$$C_w = 4I_w \frac{v^2}{r.R} = 4 \times 16 \times \frac{v^2}{0.45 \times 250} = 0.569v^2$$

$$C_m = 2I_m G\omega_w\omega_p \quad \text{(as there are two motors)}$$

$$= 2 \times 16 \times 3 \times \frac{v^2}{0.45 \times 250}$$

$$= 0.853\, v^2$$

$$C_G = C_w - C_m$$

(motors rotate in opposite direction)

$$= 0.569\, v^2 - 0.853\, v^2 = -0.284v^2$$

Reaction on each outer wheel,

$$R_{Go} = \frac{C_G}{2w} = \frac{0.284v^2}{2 \times 1.4} = 0.1014v^2 \text{ (downwards)}$$

Reaction on each inner wheel, $R_{Gi} = 0.1014\, v^2$ (upwards)

(iii) Reaction due to centrifugal couple

$$C_c = \frac{mv^2}{R}h = 3000 \times \frac{v^2}{250} \times 1 = 12v^2$$

$$R_{Co} = \frac{C_c}{2w} = \frac{12v^2}{2 \times 1.4} = 4.286v^2 \text{ (upwards)}$$

$$R_{Ci} = \frac{C_c}{2w} = 4.286\, v^2 \text{ (downwards)}$$

Total reaction on outer wheel

$= 7357.5 - 0.1014\, v^2 + 4.286v^2$

$= 7357.5 + 4.1846v^2$

Total reaction on inner wheel

$= 7357.5 - 0.1014\, v^2 - 4.286v^2$

$= 7357.5 - 4.1846v^2$

Thus, the reaction on the outer wheel is always positive (upwards). There are chances that the inner wheels leave the rails.

For maximum speed, $7357.5 - 4.1846v^2 = 0$

or $\quad v^2 = 1758.2$

$v = 41.93$ m/s

or $\quad v = \frac{41.93 \times 3600}{1000} = \underline{151 \text{ km/h}}$

Example 17.8 *A 2.2-tonne racing car has a wheel base of 2.4 m and a track of 1.4 m. The centre of mass of the car lies at 0.6 m above the ground and 1.4 m from the rear axle. The equivalent mass of engine parts is 140 kg with a radius of gyration of 150 mm. The back axle ratio is 5. The engine shaft and flywheel rotate clockwise when viewed from the front. Each wheel has a diameter of 0.8 m and a moment of inertia of 0.7 kg.m².*

Determine the load distribution on the wheels when the car is rounding a curve of 100 m radius at a speed of 72 km/h to the (i) left, and (ii) right.

Solution:

$M = 2200$ kg $\quad m = 140$ kg

$w = 1.4$ m $\quad k = 0.15$ m

$b = 2.4$ m $\quad I_w = 0.7 \text{ kg.m}^2$

$r = 0.8/2 = 0.4$ m

$R = 100$ m $\quad v = \frac{72 \times 1000}{3600} = $ m/s

(i) Car turning left (Refer Fig. 17.10)

(a) *Reaction due to weight*

Total weight = 2200 × 9.81 = 21582 N

$$R_{w1,2} = \left(21\ 582 \times \frac{1.4}{2.4}\right) \times \frac{1}{2} = 6295 \text{ N (upwards)}$$

$$R_{w3,4} = \left(21\ 582 \times \frac{1}{2.4}\right) \times \frac{1}{2} = 4496 \text{ N (upwards)}$$

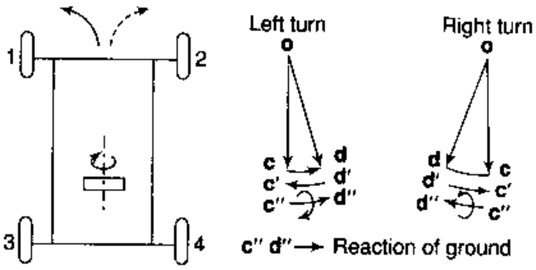


Fig. 17.10

(b) *Reaction due to gyroscopic couples*

$$C_w = 4I_w \frac{v^2}{rR} = 4 \times 0.7 \times \frac{(20)^2}{0.4 \times 100} = 28 \text{ N.m}$$

For outer wheels,

$$R'_{G2,4} = \frac{C_w}{2w} = \frac{28}{2 \times 1.4} = 10 \text{ N (upwards)}$$

For inner wheels, $R'_{G1,3} = 10$ N (downwards)

$I_e = mk^2 = 140 \times (0.15)^2 = 3.15$ kg.m^2

$$C_e = I_e G \omega_w \omega_p = 3.15 \times 5 \times \frac{(20)^2}{0.4 \times 100} = 157.5 \text{ N}$$

For front wheels,

$$R''_{G1,2} = \frac{C_e}{2b} = \frac{157.5}{2 \times 2.4} = 32.8 \text{ N (upwards)}$$

For rear wheels, $R''_{G3,4} = 32.8$ N (downwards)

(c) *Reaction due to centrifugal couple*:

$$C_c = M \frac{v^2}{R} h = 2200 \times \frac{(20)^2}{100} \times 0.6 = 5280 \text{ N.m}$$

For outer wheels,

$$R_{c2,4} = \frac{C_c}{2w} = \frac{5280}{2 \times 1.4} = 1886 \text{ N (upwards)}$$

For rear wheels, $R_{c1,3} = 1886$ N (downwards)

Therefore, reaction on wheels:

$R = R_w + R'_G + R''_G + R_c$

$R_1 = 6295 - 10 + 32.8 - 1886 = 4431.8$ N

$R_2 = 6295 + 10 + 32.8 + 1886 = 8223.8$ N

$R_3 = 4496 - 10 - 32.8 - 1886 = 2567.2$ N

$R_4 = 4496 + 10 - 32.8 + 1886 = 6359.2$ N

(ii) *Car turning right*:

All the reactions due to gyroscopic couples and centrifugal couple change signs. Therefore,

$R_1 = 6295 + 10 - 32.8 + 1886 = 8158.2$ N

$R_2 = 6295 - 10 - 32.8 - 1886 = 4366.2$ N

$R_3 = 4496 + 10 + 32.8 + 1886 = 6424.8$ N

$R_4 = 4496 - 10 + 32.8 - 1886 = 2632.8$ N

Example 17.9 *The total mass of a four-wheeled trolley car is 1800 kg. The car runs on rails of 1.6 - m gauge and rounds a curve of 24-m radius at 36 km/h. The track is banked at 10°. The external diameter of the wheels is 600 mm and each pair with axle has a mass of 180 kg with a radius of gyration of 240 mm. The height of the centre of mass of the car above the wheel base is 950 mm. Determine the pressure on each rail allowing for centrifugal force and gyroscopic couple actions.*

Solution

$M = 1800$ kg	$\theta = 10°$
$w = 1.6$ m	$h = 0.95$
$R = 24$ m	$r = 0.3$ m
$m = 180$ m	$k = 0.24$ m

$$v = 36 \text{ km/h} = \frac{36 \times 1000}{3600} = 10 \text{ m/s.}$$

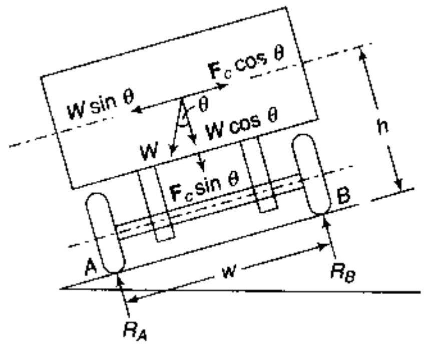


Fig. 17.11

First, considering the effect of dead weight ($W = Mg$) of the car and that of the centrifugal force on it, determine the reactions R_A and R_B at the wheels A and B (Fig. 17.11).

Resolving the forces perpendicular to the track,

$$R_A + R_B = Mg\cos\theta + \frac{Mv^2}{R}\sin\theta$$

$$= 1800 \times 9.81 \times \cos 10^\circ + \frac{1800 \times (10)^2}{24}\sin 10^\circ$$

$= 18\,692$ N

Taking moments about B,

$$R_A \times w = Mg\cos\theta\frac{w}{2} + F_c\sin\theta\frac{w}{2}$$
$$+ Mg\sin\theta \times h - F_c\cos\theta \times h$$

$$R_A = \left(Mg\cos\theta + \frac{Mv^2}{R}\sin\theta\right) \times \frac{1}{2}$$
$$+ \left(Mg\sin\theta - \frac{Mv^2}{R}\cos\theta\right) \times \frac{h}{w}$$

$$R_A = \begin{pmatrix} 1800 \times 9.81 \times \cos 10^\circ \\ + \dfrac{1800 \times (10)^2}{24} \times \sin 10^\circ \end{pmatrix} \times \frac{1}{2}$$
$$+ \begin{pmatrix} 1800 \times 9.81 \times \sin 10^\circ \\ - \dfrac{1800 \times (10)^2}{24} \times \cos 10^\circ \end{pmatrix} \times \frac{0.95}{1.6}$$

$= 9346 - 2565 = 6781$ N

$R_B = 18\,692 - 6781 = 11\,911$ N

Reaction due to cyroscopic couple

$$C_w = 2I_w\omega_w\cos\theta \times \omega_p = 2mk^2\frac{v^2}{rR} \times \cos\theta$$

$$= 2 \times 180 \times (0.24)^2 \times \frac{(10)^2}{0.3 \times 24} \times \cos 10^\circ$$

$= 283.6$ N.m

Reaction on each outer wheel,

$$R_{Go} = \frac{C_w}{2w} = \frac{283.6}{2 \times 1.6} = 88.6 \text{ N (upwards)}$$

Reaction on each inner wheels,

$R_{Gi} = 88.6$ N (downwards)

Therefore,

Pressure on outer rails $= R_B + R_{Go}$
$= 11\,911 + 88.6$
$= 11\,999.6$ N (downwards)

Pressure on inner rails $= R_A - R_{Gi}$
$= 6781 - 88.6$
$= 6692.4$ N (upwards)

(pressure is opposite to the reactions)

17.7 STABILITY OF A TWO-WHEEL VEHICLE

The case of a two-wheel vehicle can be taken in the same way as that of an automobile. However, it is easier to tilt such a vehicle inwards to neutralise the overturning effect and the vehicle can stay in equilibrium while taking a turn.

Let a vehicle take a left turn as shown in Fig. 17.12(a). The vehicle is inclined to the vertical (inwards) for equilibrium. The angle of inclination of the vehicle to the vertical is known as the *angle of heel*. Let

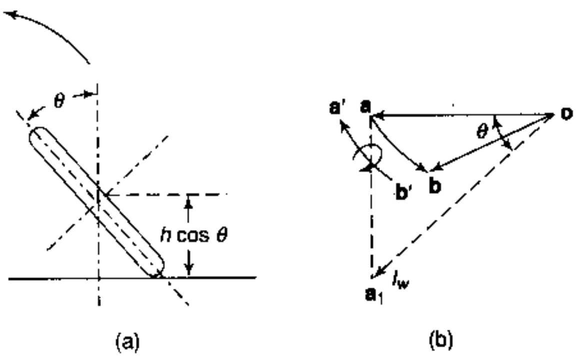


Fig. 17.12

v = linear velocity of vehicle on the track
r = radius of the wheel
R = radius of the track
I_w = moment of inertia of each wheel
I_e = moment of inertia of rotating parts of the engine
m = total mass of the vehicle and the rider

ω_w = angular velocity of the wheels

ω_e = angular velocity of rotating parts of the engine

G = gear ratio

h = height of centre of mass of the vehicle and the rider

θ = inclination of vehicle to the vertical (angle of heel)

As the axis of spin is not horizontal but inclined to the vertical at an angle θ and the axis of precession is vertical, it is necessary to take the horizontal component of the spin vector.

Spin vector (horizontal) $= I_w \cos\theta$ [See Fig. 17.12(b)]

$$= (2I_w\omega_w + I_e\omega_e)\cos\theta$$

and gyroscopic couple $= (2I_w\omega_w + I_eG\omega_w)\cos\theta\,\omega_p$

$$= (2I_w + GI_e)\frac{v}{r}\frac{v}{R}\cos\theta$$

$$= (2I_w + GI_e)\frac{v^2}{rR}\cos\theta$$

The reaction couple **b′a′** is clockwise when viewed for the rear (back) of the vehicle and tends to overturn it in the outward direction.

$$\text{Overturning couple due to centrifugal force} = \left(m\frac{v^2}{R}\right)h\cos\theta$$

$$\therefore \quad \text{total overturning couple} = (2I_w + GI_e)\frac{v^2}{rR}\cos\theta + m\frac{v^2}{R}h\cos\theta$$

$$= \frac{v^2}{R}\left(\frac{2I_w + GI_e}{r} + mh\right)\cos\theta$$

Rightening (balancing) couple due to the weight of the vehicle $= mg\,h\sin\theta$

$$\text{For equilibrium,} \quad \frac{v^2}{R}\left[\frac{2I_w + GI_e}{r} + mh\right]\cos\theta = mg\,h\sin\theta \qquad (17.8)$$

From this relation, the angle of heel θ can be determined to avoid skidding of the vehicle.

Example 17.10 *Each wheel of a motorcycle is of 600-mm diameter and has a moment of inertia of 1.2 kg.m². The total mass of the motorcycle and the rider is 180 kg and the combined centre of mass is 580 mm above the ground level when the motor cycle is upright. The moment of inertia of the rotating parts of the engine is 0.2 kg.m². The engine speed is 5 times the speed of the wheels and is in the same sense. Determine the angle of heel necessary when the motorcycle takes a turn of 35 m radius at a speed of 54 km/h.*

Solution

$m = 180$ kg $\quad I_w = 1.2$ kg m^2

$r = 0.3$ m $\quad I_e = 0.2$ kg m^2

$R = 35$ m $\quad v = \dfrac{54\times1000}{3600} = 15$ m/s

$G = \dfrac{\omega_e}{\omega_w} = 5 \quad h = 0.58$ m

Gyroscopic couple, $C_G = (2I_w + GI_e)\dfrac{v^2}{rR}\cos\theta$

$$= (2\times1.2 + 5\times0.2)\times\frac{(15)^2}{0.3\times35}\times\cos\theta$$

$= 72.86 \cos\theta$

Centrifugal couple, $C_c = m\dfrac{v^2}{R}h\cos\theta$

$$= 180\times\frac{(15)^2}{35}\times0.58\times\cos\theta$$

$= 671.14 \cos\theta$

Total overturning couple = (72.86 + 671.14) cos $\theta = 744 \cos\theta$

Rightening couple $= mg\,h \sin\theta$

$= 180 \times 9.81 \times 0.58 \sin\theta$

$= 1024 \sin\theta$

$\therefore \quad = 1025 \sin\theta = 744 \cos\theta$

or $\quad \tan\theta = \dfrac{744}{1024} = 0.727$

or $\quad \theta = \underline{36}°$

DISC AT AN ANGLE FIXED TO A ROTATING SHAFT

Consider a circular disc fixed rigidly to a rotating shaft in such a way that the polar axis of the shaft makes angle θ with the axis of the shaft (Fig. 17.13). Assume that the shaft rotates clockwise with angular velocity ω when viewed along the left end of the shaft.

Let

- OX be the axis of the shaft
- OP be the polar axis of the disc and
- OD the horizontal diametral axis of the disc

Also, let m, r and t be the mass, radius and the thickness of the disc.

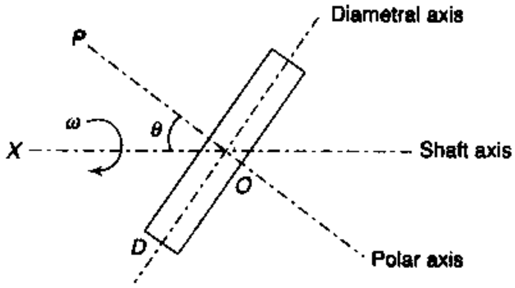


Fig. 17.13

Then I_p = moment of inertia of disc about polar axis $OP = \dfrac{m.r^2}{2}$

I_d = moment of inertia of disc about diametral axis

$$= m\left(\frac{t^2}{12} + \frac{r^2}{4}\right)$$

$$= \frac{m.r^2}{4} \qquad \text{(if the disc is thin and } t \text{ is neglected)}$$

(i) First consider the spinning about the polar axis [Fig. 17.14(a)].

Angular velocity of spin = Angular velocity of disc about the polar axis $OP = \omega\cos\theta$

Angular velocity of precession = Angular velocity of disc about the diametric axis $OD = \omega \sin\theta$

$\therefore$ gyroscopic couple $= I_p \times \omega\cos\theta \times \omega\sin\theta$

$$= \frac{1}{2} I_p \omega^2 \sin 2\theta$$

Its effect is to rotate the disc counter-clockwise when viewing from the top.

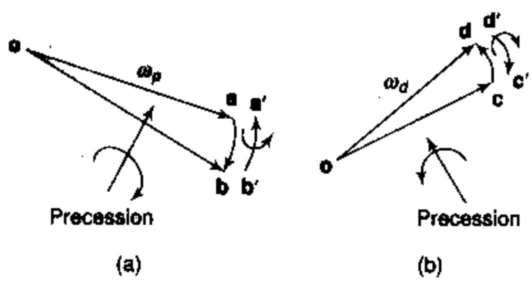


Fig. 17.14

(ii) Now consider the spinning about the diametral axis [Fig. 17.14(b)].

Angular velocity of spin = Angular velocity of disc about the diametral axis $OD = \omega\sin\theta$

Angular velocity of precession = Angular velocity of disc about the polar axis $OP = \omega\cos\theta$

$\therefore$ gyroscopic couple $= I_d \times \omega\sin\theta \times \omega\cos\theta = \dfrac{1}{2} I_d \omega^2 \sin 2\theta$

Its effect is to rotate the disc clockwise when viewing from the top.

(Angular velocity of precession is counter-clockwise when viewing from the right end along *OP*.)

Resultant gyroscopic couple on the disc, $C = \frac{1}{2}(I_p - I_d)\omega^2 \sin 2\theta$

$$= \frac{1}{2}\left(\frac{mr^2}{2} - \frac{mr^2}{4}\right)\omega^2 \sin 2\theta$$

$$= \frac{mr^2}{8}\omega^2 \sin 2\theta$$

Example 17.11 ***A uniform disc of 50-kg mass and 800 - mm diameter is mounted on a shaft. The plane of the disc is not perfectly at right angles to the axis of the shaft but has an error of 1.5 degree. Determine the gyroscopic couple acting on the bearing if the shaft rotates at 840 rpm.***

Solution

$$\omega = \frac{2\pi \times 840}{60} = 88 \text{ rad/s}$$

$$C = \frac{mr^2}{8}\omega^2 \sin 2\theta = \frac{50 \times 0.4^2}{8} \times 88^2 \sin 3^\circ$$

$= 405.3$ N.m

Summary

1. The angular velocity is represented by a vector by drawing the vector parallel to the axis of the rotor and representing the magnitude by the length of the vector to some scale. Sense of rotation of the rotor is denoted by the rule of a right-handed screw, i.e., if a screw is rotated in the clockwise direction, it goes away from the viewer and vice-versa.
2. The axis of spin, the axis of precession and the axis of gyroscopic couple are in three perpendicular planes.
3. The torque required to cause the axis of spin to precess in a plane is known as the *active gyroscopic torque* or the *applied torque*.
4. A *reactive gyroscopic torque* or *reaction torque* tends to rotate the axis of spin in the opposite direction.
5. The effect of the gyroscopic couple on a rotating body is known as the gyroscope effect on the body.
6. A gyroscope is a spinning body which is free to move in other directions under the action of external forces.
7. A four-wheel vehicle tends to turn outwards when taking a turn due to the effects of gyroscopic couple and the centrifugal force.
8. A two-wheel vehicle stabilises itself by tilting towards inside while taking a turn to nullify the effects of gyroscopic couple and the centrifugal force.

Exercises

1. In what way can the angular velocity be represented by a vector?
2. What do you mean by gyroscopic couple? Derive a relation for its magnitude.
3. What do you mean by spin, precession and gyroscopic planes?
4. Explain what is meant by applied torque and reaction torque.
5. Explain in what way the gyroscopic couple affects the motion of an aircraft while taking a turn.
6. Discuss the gyroscopic effect on sea vessels.
7. Explain the gyroscopic effect on four-wheeled vehicles.
8. What is the effect of the gyroscopic couple on the stability of a four wheeler while negotiating a curve? In what way does this effect along with that of the centrifugal force limit the speed of the vehicle?
9. How do the effects of gyroscopic couple and of

centrifugal force make the rider of a two-wheeler tilt on one side? Derive a relation for the limiting speed of the vehicle.

10. A flywheel having a mass of 20 kg and a radius of gyration of 300 mm is given a spin of 500 rpm about its axis which is horizontal. The flywheel is suspended at a point that is 250 mm from the plane of rotation of the flywheel. Find the rate of precession of the wheel. (0.52 rad/s)
11. A disc supported between two bearings on a shaft of negligible weight has a mass of 80 kg and a radius of gyration of 300 mm. The distances of the disc from the bearings are 300 mm to the right from the left-hand bearing and 450 mm to the left from the right-hand bearing. The bearings are supported by thin vertical cords. When the disc rotates at 100 rad/s in the clockwise direction looking from the left-hand bearing, the cord supporting the left-hand side bearing breaks. Find the angular velocity of precession at the instant the cord is cut and discuss the motion of the disc. (0.327 rad/s)
12. The moment of inertia of an aeroplane air screw is 20 kg.m^2 and the speed of rotation is 1000 rpm clockwise when viewed from the front. The speed of the flight is 200 km per hour. Find the gyroscopic reaction of the air screw on the aeroplane when it makes a left-handed turn on a path of 150-m radius. (775.5 N.m)
13. The rotor of a marine turbine has a moment of inertia of 750 kg.m^2 and rotates at 3000 rpm clockwise when viewed from aft. If the ship pitches with angular simple harmonic motion having a periodic time of 16 seconds and an amplitude of 0.1 radian, find the
    (i) maximum angular velocity of the rotor axis
    (ii) maximum value of the gyroscopic couple
    (iii) gyroscopic effect as the bow dips

    (0.0393 rad/s; 9261 N.m; bow swings to port (left) as it dips)
14. The turbine rotor of a sea vessel having a mass of 950 kg rotates at 1200 rpm clockwise while looking from the stern. The vessel pitches with an angular velocity of 1.2 rad/s. What will be the gyroscopic couple transmitted to the hull when the bow rises? The radius of gyration of the rotor is 300 mm. (12.89 kN.m)
15. A ship is propelled by a turbine rotor having a mass of 6 tonnes and a speed of 2400 rpm. The direction of rotation of the rotor is clockwise when viewed from the stern. The radius of gyration of the rotor is 450 mm. Determine the gyroscopic effect when the
    (i) ship steers to the left in a curve of 60 m radius at a speed of 18 knots (1 knots = 1860 m/h)
    (ii) ship pitches 7.5 degrees above and 7.5 degrees below the normal position and the bow is descending with its maximum velocity; the pitching motion is simple harmonic with a periodic time of 18 seconds
    (iii) ship rolls and at the instant, its angular velocity is 0.035 rad/s counter-clockwise when viewed from the stern

    Also, find the maximum angular acceleration during pitching.

    (47.33 kN.m, bow is raised; 13.96 kN.m, ship turns towards port side; 10. 69 kN.m, no gyroscopic effect; 0.016 rad/s^2)
16. A rear engine automobile is travelling along a curved track of 120 m radius. Each of the four wheels has a moment of inertia of 2.2 kg/m^2 and an effective diameter of 600 mm. The rotating parts of the engine have a moment of inertia of 1.25 kg.m^2. The gear ratio of the engine to the back wheel is 3.2. The engine axis is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The mass of the vehicle is 2050 kg and the centre of mass is 520 mm above the road level. The width of the track is 1.6 m. What will be the limiting speed of the vehicle if all the four wheels maintain contact with the road surface? (150.2 km/h)
17. The moment of inertia of a pair of locomotive driving wheels with the axle is 200 kg.m^2. The distance between the wheel centres is 1.6 m and the diameter of the wheel treads is 1.8 m. Due to defective ballasting, one wheel falls by 5 mm and rises again in a total time of 0.12 seconds while the locomotive travels on a level track at 100 km/h. Assuming that the displacement of the wheel takes place with simple harmonic motion, determine the gyroscopic couple produced and the reaction between the wheel and rail due to this couple. (505 N.m; 315.6 N)
18. Each road wheel of a motor cycle is of 600 mm diameter and has a moment of inertia of 1.1 kg.m^2. The motorcycle and the rider together weigh 220 kg and the combined centre of mass is 620 mm above the ground level when the motor cycle is upright. The moment of inertia of the rotating parts of the engine is 0.18 kg/m^2. The engine rotates at 4.5 times the speed of road wheels in the same sense. Find the angle of heel necessary when the motor cycle is taking a turn of 35 m radius at a speed of 72 kg/h. (38.6°)